

A REMARK ON THE APPLICATION OF VAN KAMPEN'S THEOREM

LOUIS YANG LIU

Van Kampen's Theorem is an important theorem in Algebraic Topology. It is used frequently to compute fundamental groups of topological spaces which are unions of subspaces. Moreover we actually need to use it in some sense to compute homology group, since Mayer-Vietoris theorem is a generalised version of Van Kampen's theorem. The key of applying Van Kampen's Theorem is to check whether the assumptions are satisfied in various specific situations. Let's look at an example.

Example. ([1]) Prove that the union of finitely many of convex sets with non-empty intersection for any three of them is simply connected.

In this problem, we want to compute the fundamental group of the union $\bigcup_{i=1}^n K_i$, and try to use Van Kampen's theorem. If we want to apply it directly, then we need that $\bigcap_{i=1}^n K_i$ is non-empty, but this is too strong to get. Actually the intersection of all convex sets is not necessarily non-empty though any three of them has non-empty intersection. So do not try to show the plausible statement that the intersection of all convex sets is non-empty. Just do it by induction and it is not hard to see the assumptions in Van Kampen's Theorem are satisfied in the induction.

REFERENCES

- [1] Allen Hatcher, Algebraic Topology, Cambridge University Press, 1 edition, 2001