University of Georgia   **Math 1113**
Department of Mathematics   **Final Exam**   Fall 2023

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____   Name (print): _____

Student Number: _____

Instructor's Name: _____   Class Time: _____

Problem Number	Total Points Possible	Points Made
1	0	
2	20	
3	20	
4	15	
5	15	
6	20	
7	15	
8	15	
9	10	
10	15	
11	10	
12	10	
13	10	
14	10	
Total:	185	

- If you need extra space use the last page. *Do not tear off the last page!*

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If we can't read it (or cannot find it), we cannot grade it.

- Please turn off your mobile phone.

- You are only allowed to use a TI-30 calculator. No other calculators are permitted.

- A calculator is not necessary, and answers should be given in a form that can be directly entered into a calculator. If you give a numerical value it should be to within one decimal place unless otherwise stated.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

1. [2 Bonus] Common Knowledge: What happened to Ethan Allen after the Battle of Montreal?

2. Determine all values of x that satisfy each equation below.

   (a) [5 pts] $5 = \dfrac{1}{\sqrt{x+1}}$

   (b) [5 pts] $4x^2 - 7 = x$

(c) [5 pts] $2\ln(3x - 1) + 1 = 4.5$

(d) [5 pts] $5 \cdot 7^{3+x} = 9 \cdot 3^{2x-1}$

3. Determine the value of each of the requested quantities below. Numerical values should be to within 0.01 of the true value. (**All angles are given in radians unless otherwise stated and your answer should be expressed in radians if you have to determine their numerical value**.)

  (a) [5 pts]   Determine the cosine, sine, and tangent of the angle α in the diagram below.
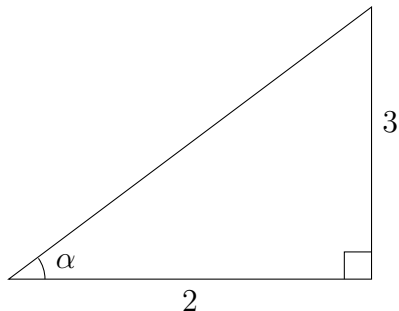
  (b) [5 pts]   Determine the exact numerical value of the following expression and express it without the use of any trigonometric functions:

$$\cos\left(\arctan\left(-\frac{2}{5}\right)\right).$$

  (Show your work and do not provide a numerical estimate or use a calculator.)

(c) [5 pts]   Determine the area of a sector whose radius is fifteen and the arclength is five.

(d) [5 pts] Determine the exact numerical value of the following expression and express it without the use of any trigonometric functions:

$$\sin\left(\arccos(0.1) + \arcsin(0.3)\right).$$

(Show your work and do not provide a numerical estimate or use a calculator.)

4. In each question below a description of a line is given. Use the information in each case to determine an equation that defines the line.

(a) [5 pts] The line whose graph includes the points $(1, 5)$ and $(-3, 4)$.

(b) [5 pts] The line whose graph includes the point $(4, -1)$ and is parallel to the line defined by $3x + 7y = 9$.

(c) [5 pts] The line that has an x-intercept of $(5, 0)$, and y increases by two when x increases by five.

5. Two quadratic functions are defined,

$$
\begin{aligned}
f(x) &= a(x-4)^2 + 2, \\
g(x) &= -2(x-4)^2 + 3,
\end{aligned}
$$

where a is a constant.

(a) [5 pts] Determine the vertex of g. Does the function g have a minimum or a maximum value at its vertex? (Provide a brief justification for your conclusion.)

(b) [5 pts] What are the possible set of values of a that will guarantee that the function $f(x)$ is increasing for all $x > 4$? (Provide a brief justification for your conclusion.)

(c) [5 pts] What are the possible set of values of a that will guarantee that the graph of $f(x)$ intersects the x-axis twice? (Provide a brief justification for your conclusion.)

6. The following questions refer to the function

$$h(x) = 5e^{x+2}.$$

(a) [5 pts] Determine the range and domain of the function h.

(b) [5 pts] Determine the inverse of h.

(c) [5 pts] Determine the range and domain of the inverse of h.

(d) [5 pts] Determine the equations for the asymptotes of both h and the inverse of h.

Asymptote(s) for h: _____

Asymptote(s) for the inverse of h: _____

7. For each scenario below circle the phrase that will best describe the **kind** of function that will best approximate the phenomena under consideration.

   (a) [5 pts] The number of fungal spores increases as a constant rate as the temperature increases. *The number of fungal spores as a function of temperature.*

Linear	Quadratic	Exponential	Trigonometric
Function	Function	Function	Function

   (b) [5 pts] The number of fungal spores changes throughout the year. It reaches a maximum in the Fall and a minimum in the Spring. It repeats the cycle each year. *The number of fungal spores as a function of time in months since the start of the year.*

Linear	Quadratic	Exponential	Trigonometric
Function	Function	Function	Function

   (c) [5 pts] The number of fungal spores released increases by 3% each hour of the day starting at sunrise. *The number of fungal spores as a function of time in hours since sunrise of the current day.*

Linear	Quadratic	Exponential	Trigonometric
Function	Function	Function	Function

8. Use the following values to provide estimates of each of the expressions below.

$$\begin{aligned} \log_a(u) &\approx 1.308, \\ \log_a(w) &\approx 1.552, \\ \log_a(b) &\approx -2.561. \end{aligned}$$

(a) [5 pts] $\log_a(u \cdot w)$

(b) [5 pts] $\log_a\left(\dfrac{u \cdot w}{b^4}\right)$

(c) [5 pts] $\log_a(u \cdot b) - \log_a(b \cdot w)$

9. [10 pts] In 1775 Ethan Allen crossed the Saint Lawrence River with a group of American forces to try and capture Montreal. General Allen knew that the walls of the city were near the shore line, and the height of the walls were 6.4 meters over the surface level of the river. When General Allen looked at the top of the wall and estimated an angle of elevation of 4.5° approximately how far was he from the walls of the city?

10. Before a new drug is tested on humans it is tested in a laboratory on an animal, and the results are used to estimate the maximum safe dosage for a human. The human equivalent dosage does not scale in a simple manner.[1] A new drug has been tested on guinea pigs, and the drug's human equivalent dosage (HED) in milligrams is estimated to be

$$\text{HED} \;=\; 0.32 \cdot \left(\frac{\text{Surface area}}{\text{Weight}} \right).$$

It is difficult to estimate the surface area of a person, so the surface area for a person is estimated by

$$\text{Surface area} \;=\; 10 \cdot (\text{Weight})^{2/3},$$

where the "Weight" is the weight of the person in kilograms.

(a) [5 pts] A person has a weight of 60 kg. What is the HED for the person?

(b) [5 pts] Determine the function that represents the weight of a person given the HED.

(c) [5 pts] It is determined that a dosage greater than 0.7 mg will increase the risks associated with the drug. What is the weight of a person where the dosage should be capped to 0.7 mg?

---

[1] A Guide for Estimating the Maximum Safe Starting Dose and Conversion it between Animals and Humans Mohamed J. Saadh, Mansour Haddad2, Moeen F. Dababneh1, Mohammad F. Bayan2, Bilal A. Al-Jaidi, Sys Rev Pharm 2020;11(8):98-101

11. [10 pts] Determine a formula for the function whose graph is shown below expressed as a sine function,

$$k(x) \;=\; A\sin(bx + c) + d.$$

The values of A and b should be positive numbers.
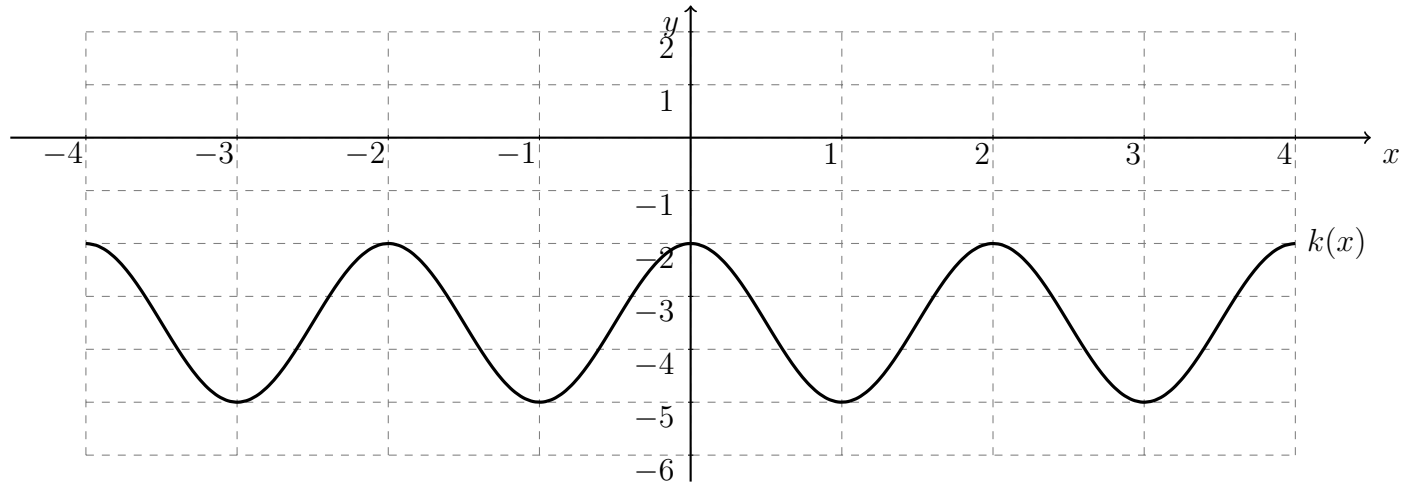
$A =$

$b =$

$c =$

$d =$

12. [10 pts] Verify the following identity,

$$\csc(\theta) - \sin(\theta) \;=\; \cot(\theta)\cos(\theta).$$

13. [10 pts] A byproduct generated in a food processing plant is accidentally released into a nearby holding pond, and the material slowly degrades. The release was not noticed, and two days after the release it is estimated that there is 350 kg of the byproduct in the holding pond. Five days after the initial release it is estimated that there is 300 kg of the material in the holding pond. According to state regulations remediation actions will be required if there is more than 275 kg of the material seven days after the initial spill. Determine the initial amount of material released assuming the amount of material decays exponentially.

14. [10 pts] A billboard will be constructed in the shape of a rectangle and will be put next to a scenic highway in a pristine part of our beautiful state. A brace will be constructed around the perimeter of the sign to help hold it upright. The brace will cost ten dollars per meter for the horizontal portion and twenty dollars per meter for the vertical portion to help stabilize the sign. A total of $500 will be spent on the brace. What dimensions will make for the largest possible area? (Show and justify all of your work and do not provide just an intuitive estimate.)

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____