By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): Solutions Name (print): __________

Student Number: __________

Instructor's Name: __________ Class Time: __________

Problem Number	Points Possible	Points Made
1	25	
2	15	
3	15	
4	5	
5	5	
6	10	
7	15	
8	15	
9	15	
10	10	
11	15	
12	15	
13	20	
14	10	
Total:	190	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a TI-30 calculator. No other calculators are permitted, and TI-30X Pro calculators are not permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

1. Determine the first derivative of each of the following functions. Print your answer in the box provided. You do not have to simplify your answers.

_______ (a) [5 pts] $g(x) = 3x^5 - \dfrac{20}{x^2} - 900$

$g'(x) =$ $15x^4 + 40x^{-3}$

_______ (b) [5 pts] $h(r) = 5r\cos(r-3) - 4r + 10$

$h'(r) =$ $5r(-\sin(r-3)) + \cos(r-3)\cdot 5 - 4$

_______ (c) [5 pts] $f(t) = \arctan(t)$

$f'(t) =$ $\dfrac{1}{1+t^2}$

(d) [5 pts] $R(x) = 5x - \dfrac{e^x}{x^2 - 1}$ $= 5x - (e^x)(x^2-1)^{-1}$

$R'(x) =$ $5 - \dfrac{(x^2-1)e^x - e^x(2x)}{(x^2-1)^2}$ $\underline{\underline{\text{OR}}}$ $5 - e^x \cdot -1(x^2-1)^{-2} \cdot 2x - (x^2-1)^{-1} \cdot e^x$

(e) [5 pts] $P(x) = 5x^2 \tan\left(\sqrt{x-1}\right)$

$P'(x) =$ $5x^2 \cdot \sec^2(\sqrt{x-1}) \cdot \frac{1}{2}(x-1)^{-1/2} + \tan(\sqrt{x-1}) \cdot 10x$

2. Determine the **the most general anti-derivative** of each of the following expressions. Print your answer in the box provided.

_______ (a) [5 pts] $10x^2 + 4x - 100$

Most general anti-derivative: $\frac{10}{3}x^3 + 2x^2 - 100x + C$

_______ (b) [5 pts] $4\sin(t) - 8$

Most general anti-derivative: $-4\cos(t) - 8t + C$

_______ (c) [5 pts] $x\cos\left(x^2 + 1\right)$

Most general anti-derivative: $\frac{1}{2}\sin(x^2+1) + C$

3. Evaluate the following definite integrals. Print your answer in the box provided.

________ (a) [5 pts] $\int_1^2 \left(e^t - t^{-1}\right) dt$

$$= \left[e^t - \ln|t|\right]_1^2$$

Value: $(e^2-\ln(2))-(e-\ln(1))$ or $e^2-e-\ln(2)$

________ (b) [5 pts] $\int_0^{\pi/8} \sec^2(2x)\ dx = \left[\frac{1}{2}\tan(2x)\right]_0^{\pi/8}$

$$= \frac{1}{2}\tan(\pi/4) - \frac{1}{2}\tan(0) = \boxed{\frac{1}{2}}$$

Value: $\frac{1}{2}\tan(2\pi/8) - \frac{1}{2}\tan(2\cdot 0)$

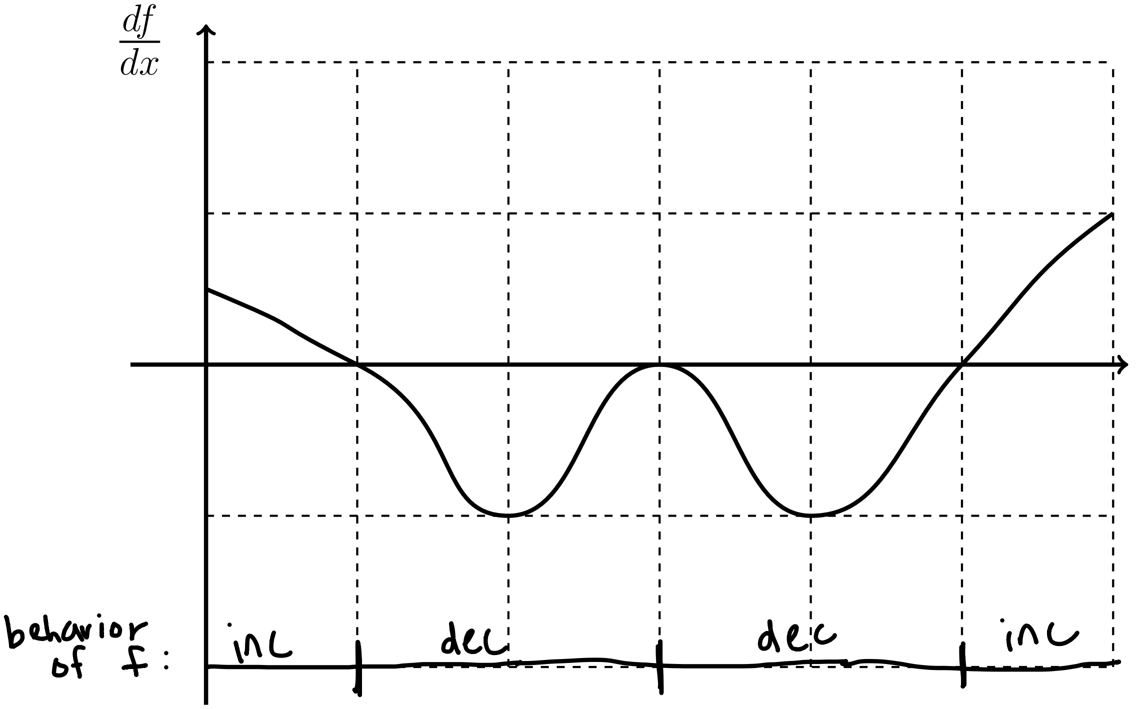
________ (c) [5 pts] $\int_0^{1/2} \frac{x}{\sqrt{1-x^4}}\, dx = \int_0^{1/2} \frac{1}{\sqrt{1-(x^2)^2}} \cdot x\, dx$

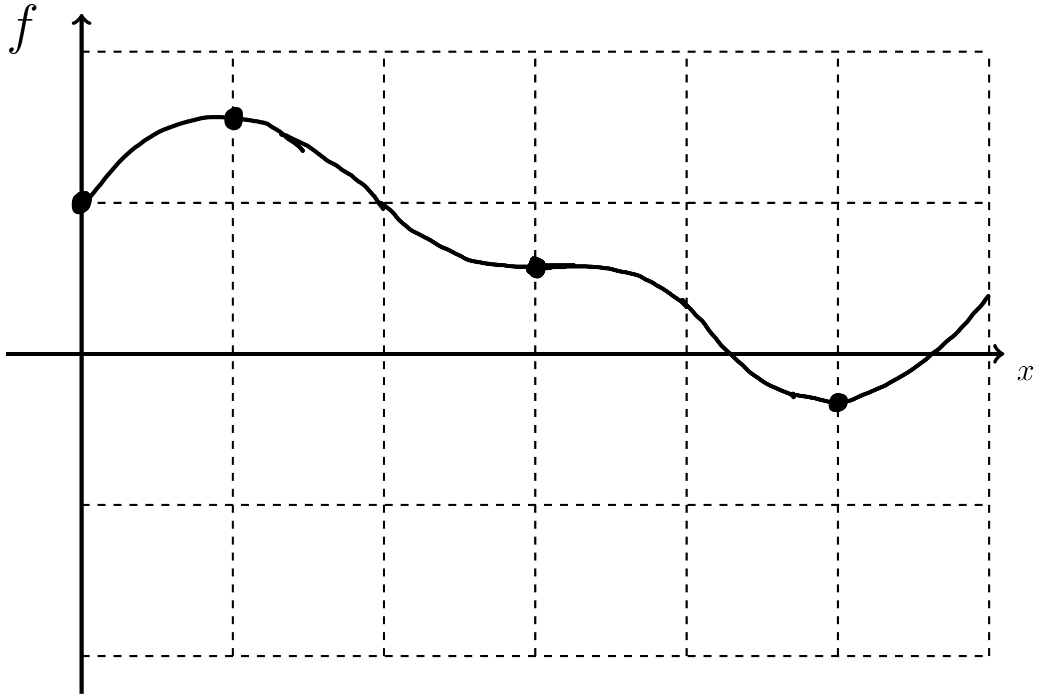
$$= \left[\frac{1}{2}\arcsin(x^2)\right]_0^{1/2} = \frac{1}{2}\arcsin(1/4) - \frac{1}{2}\arcsin(0) = \frac{1}{2}\arcsin(1/4)$$

Value: $\frac{1}{2}\arcsin(1/4) - \frac{1}{2}\arcsin(0)$

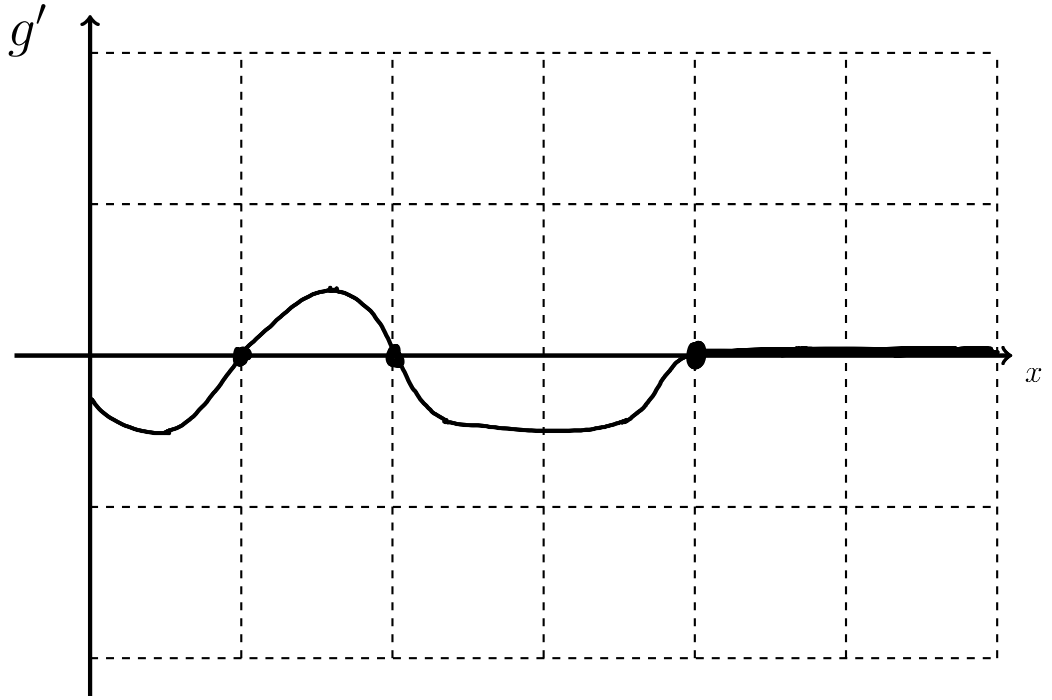
4. [5 pts] Make a sketch of $f(x)$ given that $f(0) = 1$, where the graph of $\dfrac{df}{dx}$ is shown below.

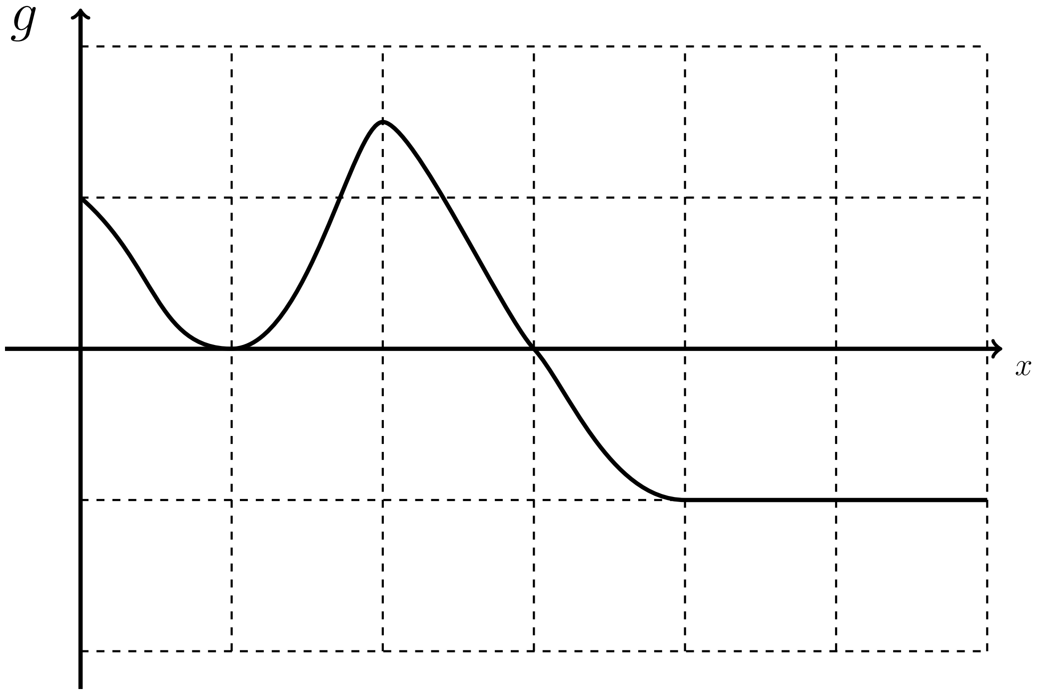
________ (The grid lines are one unit apart.)





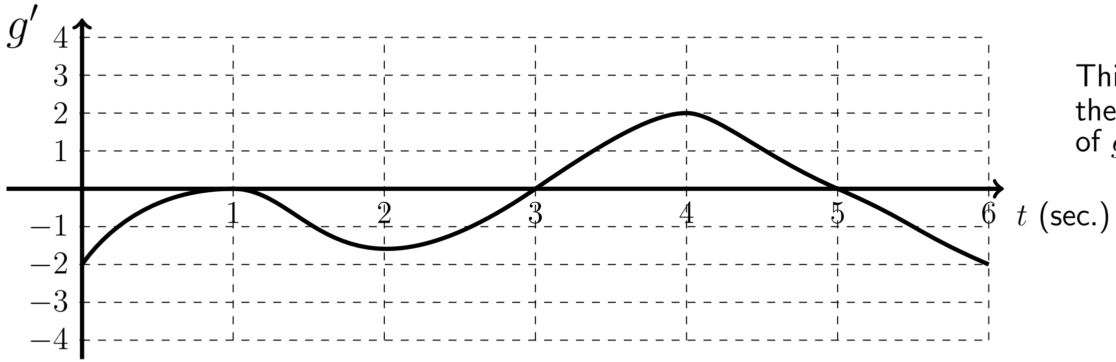
5. [5 pts] Make a sketch of $g'(x)$ where the graph of g is shown below. (The grid lines are one unit apart.)



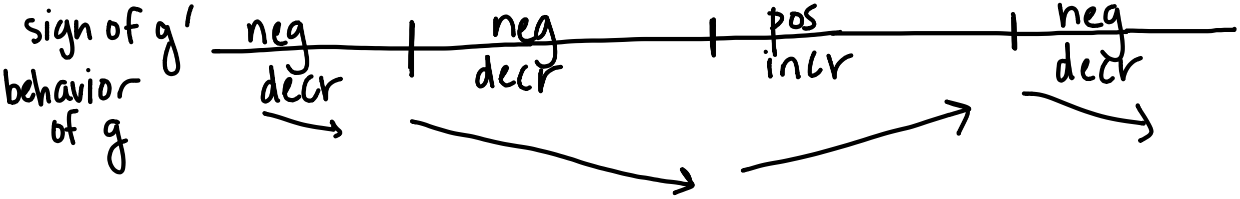


6. The derivatives of two different functions are described in parts (a) and (b).

   (a) [5 pts] The graph of the derivative of a function, g, is given below. Determine all values of t where a local maximum or a local minimum occurs for the original function. **Provide a brief justification for each value of t you give.**

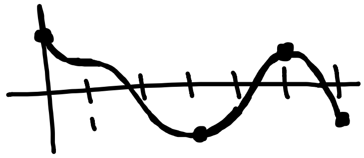


This is the derivative of g.



Based on increasing/decreasing behavior, g has local minima at $t=3$ and $t=6$ and local maxima at $t=0$ and $t=5$

Sketch of graph of $y=g(x)$:



Points earned: ____
out of a possible 5 points

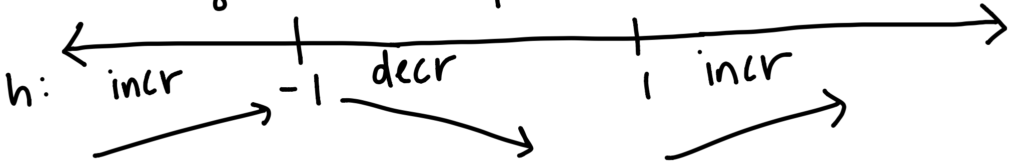
(b) [5 pts] The derivative of a function, h, is given by

$$h'(x) = \frac{(x^2-1)}{x^2+4}$$

where x is any real number. Determine all values of x where a local maximum or a local minimum occurs for the original function. **Give a brief justification for each value of x.**

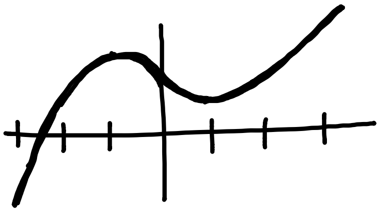
critical numbers: $x = \pm 1$

$h'(-2) = \frac{3}{8}$ $h'(0) = \frac{-1}{4}$ $h'(2) = \frac{3}{8}$



Due to increasing/decreasing behavior, h has a local max at $x=-1$ and a local min at $x=1$.

Sketch of graph of $y=h(x)$:



7. A car moves along a straight track. The car starts at the origin and has an initial velocity of 2 m/sec. The car's acceleration is

$$a(t) \quad = \quad -6e^{-3t} \quad \text{m/sec}^2,$$

where t is the time in seconds and $t \geq 0$.

______ (a) [12 pts] Determine the position of the car at time t where $t \geq 0$.

$v(t) = 2e^{-3t} + C$

$2 = v(0) = 2e^0 + C$

$2 = 2 + C$

$C = 0$

$v(t) = 2e^{-3t}$

$s(t) = \frac{-2}{3}e^{-3t} + D$

$0 = s(0) = \frac{-2}{3}e^0 + D = \frac{-2}{3} + D$

$D = 2/3$

$s(t) = \boxed{\frac{-2}{3}e^{-3t} + \frac{2}{3}}$

(b) [3 pts] What is the car's position after a very long time?

______ $\lim_{t \to \infty} s(t) = \frac{-2}{3} \cdot 0 + \frac{2}{3} = \boxed{\frac{2}{3}}$ meters

8. Information about two functions and their derivatives are given in the tables below. Use the information to determine the values of each of the quantities below.

t	$f(t)$
0	4
1	-8
2	3
3	1

t	$f'(t)$
0	7
1	-6
2	4
3	0

t	$g(t)$
0	2
1	-3
2	5
3	-7

t	$g'(t)$
0	3
1	8
2	-2
3	-1

(a) [5 pts] $\frac{d}{dt}(2f(t) - 4g(t))$ at $t = 1$.

$= 2f'(1) - 4g'(1)$

$= 2(-6) - 4 \cdot 8$

$= -12 - 32$

$= \boxed{-44}$

(b) [5 pts] $\frac{d}{dt}(2f(t) \cdot g(t))$ at $t = 3$.

$= 2f(3) \cdot g'(3) + 2f'(3) \cdot g(3)$

$= 2(1)(-1) + 2(0)(-7)$

$= \boxed{-2}$

(c) [5 pts] $\frac{d}{dt}\Big(-g(f(t))\Big)$ at $t = 2$.

$= -g'(f(2)) \cdot f'(2)$

$= -g'(3) \cdot 4$

$= -(-1) \cdot 4$

$= \boxed{4}$

9. Answer the questions below for the graph of the equation $y = 2\cos(y - \pi x)$.

(a) [5 pts] Determine $\frac{dy}{dx}$.

$$\frac{dy}{dx} = -2\sin(y-\pi x)\cdot\left(\frac{dy}{dx}-\pi\right)$$

$$\frac{dy}{dx} = -2\sin(y-\pi x)\frac{dy}{dx} + 2\sin(y-\pi x)\cdot\pi$$

$$\frac{dy}{dx} + 2\sin(y-\pi x)\frac{dy}{dx} = 2\pi\sin(y-\pi x)$$

$$\frac{dy}{dx}\left(1+2\sin(y-\pi x)\right) = 2\pi\sin(y-\pi x)$$

$$\boxed{\frac{dy}{dx} = \frac{2\pi\sin(y-\pi x)}{1+2\sin(y-\pi x)}}$$

(b) [5 pts] Determine an equation of the tangent line to the curve at the point $\left(\frac{1}{2}, 0\right)$.

$$\left.\frac{dy}{dx}\right|_{(\frac{1}{2},0)} = \frac{2\pi\sin(0-\pi/2)}{1+2\sin(0-\pi/2)} = \frac{-2\pi}{1+-2} = 2\pi$$

$$\boxed{y-0 = 2\pi\left(x-\frac{1}{2}\right)}$$

or $y = 2\pi x - \pi$

(c) [5 pts] Find a point, $(x, 2)$, on the curve where the tangent line is horizontal.

$0 = 2\pi\sin(2-\pi x)$

$0 = \sin(2-\pi x)$

$k\pi = 2-\pi x$, k any integer

$k\pi - 2 = -\pi x$

$x = \frac{2-k\pi}{\pi}$ back to curve!

$2 = 2\cos(2-\pi x)$

$1 = \cos(2-\pi x)$

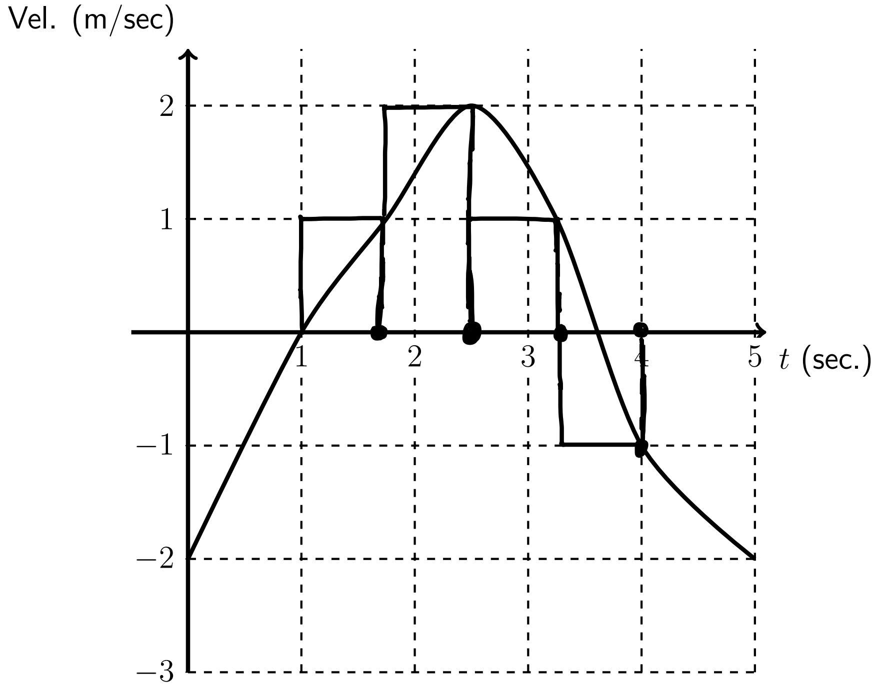
$2-\pi x = 2\ell\pi$, ℓ any integer

$2-2\ell\pi = \pi x$

$x = \frac{2-2\ell\pi}{\pi}$

Find one x that works in $x = \frac{2-k\pi}{\pi}$ and $x = \frac{2-2\ell\pi}{\pi}$. One correct answer is $x = \frac{2-0}{\pi} = 2/\pi$ (but there are other correct answers).

10. The graph of an object's velocity is given in the plot below.



________ (a) [7 pts] Approximate the integral of the velocity from $t = 1$ second to $t = 4$ seconds using a Riemann sum with right endpoints and four equally spaced sub-intervals. (You do not have to simplify your answer.)

$\Delta x = \frac{4-1}{4} = \frac{3}{4}$

$[1, \overset{*}{7/4}]\ [7/4, \overset{*}{10/4}]\ [10/4, \overset{*}{13/4}]\ [13/4, \overset{4}{\overset{\downarrow}{16/4}}]$ *

$f(7/4) \cdot 3/4 + f(10/4) \cdot 3/4 + f(13/4) \cdot 3/4 + f(4) \cdot 3/4$

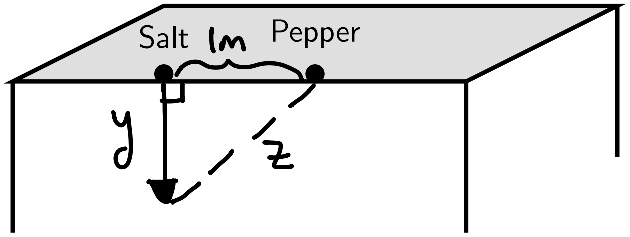
$= 1 \cdot \frac{3}{4} + 2 \cdot \frac{3}{4} + 1 \cdot \frac{3}{4} - 1 \cdot \frac{3}{4} = \frac{9}{4}$

________ (b) [3 pts] What does the number calculated above represent for the object?

It's an estimated displacement for the object from $t=1$ to $t=4$.

11. [15 pts] Salt and Pepper sit on a table, and they are 1 meter apart from each other. The top of the table is 0.8 meters above the hard tile floor, and Salt is near the edge of the table. A cat jumps on the table and gently pushes Salt over the edge. Salt's vertical velocity is $v(t) = -3t$ m/sec., where t is the time after being knocked over, and the negative direction is downward toward the floor.

At what rate is the distance between Salt and Pepper changing at any time after Salt falls off the edge of the counter and before Salt hits the floor?



goal: $\frac{dz}{dt}$

$y'(t) = -v'(t) = 3t \leftarrow$ y is an increasing quantity

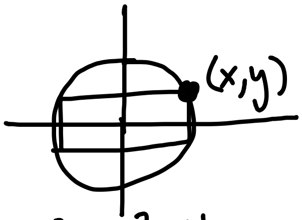
$$\left.\begin{array}{l} y(t) = \frac{3}{2}t^2 + C \\ y(0) = 0 \end{array}\right\} y(t) = \frac{3}{2}t^2$$

$$1^2 + y^2 = z^2$$

$$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt} = \boxed{\frac{(3/2\,t^2)}{\sqrt{1+(\frac{3}{2}t^2)^2}}\,(3t)}$$

12. [15 pts] A rectangle is inscribed in a circle of radius two centered at the origin. Determine the dimensions of the rectangle with the largest possible area, and determine the largest possible area. (Use calculus to show that the dimensions really give the largest possible area.)



$x^2+y^2=4$

goal: max area

$A = 2x \cdot 2y$

$A = 4x\sqrt{4-x^2}$

domain: $[0,2]$ or $(0,2)$

$A' = 4x \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot -2x + \sqrt{4-x^2} \cdot 4$

$A' = \frac{-4x^2}{\sqrt{4-x^2}} + \frac{4(\sqrt{4-x^2})^2}{\sqrt{4-x^2}} = \frac{-4x^2+16-4x^2}{\sqrt{4-x^2}} = \frac{16-8x^2}{\sqrt{4-x^2}}$

critical numbers: $16-8x^2=0$

$2 = x^2$

$x = \sqrt{2}$

using $[0,2]$:

x	A
0	0
$\sqrt{2}$	$4\sqrt{2}\cdot\sqrt{2} = 8$ ↙ max area
2	0

using $(0,2)$:

$(0,\sqrt{2})$ $(\sqrt{2},2)$

$A'(1) = \frac{8}{\sqrt{3}}$ $A'(1.5) \approx -1.512$

↗ ↘

local max, only one CN so it's an absolute max

The largest possible area is $A(\sqrt{2}) = 8$, and the dimensions are $2x = 2\sqrt{2}$ and $2y = 2\sqrt{4-(\sqrt{2})^2} = 2\sqrt{2}$ inches.

13. Determine the values of each of the limits given below.

(a) [5 pts] $\lim_{x\to 5}\left(x^3-7x+10\right)$

$$=5^3-7\cdot 5+10$$
$$=125-35+10$$
$$=100$$

(b) [6 pts] $\lim_{x\to-\infty}\frac{7|x|-3}{4x-12}=\lim_{x\to-\infty}\frac{7(-x)-3}{4x-12}\overset{LH}{=}\lim_{x\to-\infty}\frac{-7}{4}=\boxed{\frac{-7}{4}}$

or $\lim_{x\to\infty}\left(\frac{7\left(\frac{|x|}{x}\right)-\frac{3}{x}}{4-\frac{12}{x}}\right)=\frac{7\cdot -1-0}{4-0}=\frac{-7}{4}$

(c) [9 pts] $\lim_{x\to 0^+}(\sin(x))^{8x}\overset{0^0}{=}e^0=\boxed{1}$

$$\lim_{x\to 0^+}\ln\left[(\sin(x))^{8x}\right]=\lim_{x\to 0^+}8x\ln(\sin(x))=\lim_{x\to 0^+}\frac{8\ln(\sin(x))}{x^{-1}}\qquad\frac{-\infty}{\infty}$$

$$\overset{LH}{=}\lim_{x\to 0^+}\frac{8\cdot\frac{1}{\sin(x)}\cdot\cos(x)}{-x^{-2}}=\lim_{x\to 0^+}-8\cdot\frac{\cos(x)}{\sin(x)}\cdot x^2=\lim_{x\to 0^+}\left(-8\cos(x)\cdot x\right)\cdot\left(\frac{x}{\sin(x)}\right)$$

$=-8\cdot 1\cdot 0\cdot 1=0$ go back to the original → $e^0=\boxed{1}$

14. The power in an electrical circuit is given by

$$P = I^2 R,$$

where P is the power (Watts), I is the current (Amps) and R is the resistance (Ohms) in the circuit. The circuit will be designed to have a constant resistance of $R = 5,000$ Ohms.

(a) [5 pts] Determine the linearization of the power in terms of the current for a level of 0.1 Amps of current.

$P = 5000 I^2$

$I = 0.1$ amps

$f(a) = 5000(0.1)^2 = 50$ -or- $P(0.1) = 50$

$f'(a) = 10\,000(a) = 1000$ $\quad P'(0.1) = 1000$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = 50 + 1000(x - 0.1)$ or $P \approx 50 + 1000(I - 0.1)$

(b) [5 pts] For the final design the power should be about 50 Watts, but it can vary by ±0.1 Watts. Use the linearization or the differential to estimate the possible change in the current.

dP at most 0.1

$dP = 10000 I \, dI$

$dI = \frac{dP}{10,000 I} \approx \frac{0.1}{10,000(0.1)}$

$dI \approx \boxed{.0001}$ amps

or

$\underbrace{P - 50}_{0.1} \approx 1000 \underbrace{(I - 0.1)}_{\Delta I}$

$\Delta I \approx \frac{0.1}{1000} = \boxed{.0001}$ amps

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): __________ Instructor (print): __________ Time: __________

Points earned: _____
out of a possible 0 points