PRINTED NAME : _____

STUDENT ID : _____

DATE : ____/____/____

GRADE

/245

INSTRUCTOR : _____   SECTION : _____

Nº	SCORE	MAX
1		30
2		5
3		5
4		5
5		5
6		5
7		5
8		5
9		5
10		5
11		5
12		5
13		5
14		5
15		5
16		5
17		10
18		10
19		10
20		10
21		10
22		10
23		10
24		10
25		10
26		10
27		10
28		10
29		10
30		10
TOTAL		245

INSTRUCTIONS

- This is a set of practice problems that can be used to review for the final exam. The real exam follows the same format, but is shorter.

- The exam has two parts: the first one consists of Multiple Choice (MC) questions, and the second part of Free Response (FR) ones. You <u>must</u> show work for both parts. An unjustified answer will receive no credit. If you are using a shortcut, explain it.

- Your work must be neat and organized. Circle the answer for MC questions and put a box around the final answer for the FR questions. There is only one correct answer for each MC question.

- Smart devices (including smart watches and cell phones) are not allowed and may not be on your person.

- If you plan to use a calculator, you are only allowed to use a TI-30XS Multiview calculator; the name must match exactly. No other calculators or sharing of calculators is allowed. Include an exact answer for each problem. Answers containing symbolic expressions such as $\cos(3)$ and $\ln(2)$ are perfectly acceptable.

- If you need extra space, use the last page. Any solution that is without indication on the scrap paper and not in the designated space, will not be graded.

Part I: Multiple Choice

Show your work and **circle** your answer.

1. [30 pts] Find the following limits. If they do not exist, choose DNE.

_____PTS   (a) $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{\cos(x) + \cos(2x)}$

      (A) DNE   (C) 3   (E) -3

      (B) 0   (D) -1

_____PTS   (b) $\displaystyle\lim_{x\to 2^-} \frac{x^2 - 5x + 6}{x - 2}$

      (A) DNE   (C) -1   (E) 4

      (B) 0   (D) 1

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(e) $\displaystyle\lim_{x\to 0^+} (x + \pi/x)^{\pi x}$

      (A) $-\infty$ (C) 1 (E) DNE

      (B) 0 (D) ∞

(f) $\displaystyle\lim_{x\to +\infty} \sqrt{x^2 - x} - x$

      (A) $-\infty$ (D) 0 (G) DNE

      (B) -1 (E) $\frac{1}{2}$

      (C) $-\frac{1}{2}$ (F) ∞

_____PTS 2. [5 pts] Assume that $x = a$ is a vertical asymptote of $y = \dfrac{x^2 - 25}{(x-2)(x^2 - 9x + 20)}$. Find the sum of all possible values of a:

(A) -5 (C) 5 (E) 10

(B) 6 (D) 8

_____PTS 3. [5 pts] The graph of $y = \dfrac{x - 5}{x^2 - 7x + a}$ has a vertical asymptote and a removable discontinuity. What is the value of a?

(A) 0 (D) 7

(B) 3 (E) 10

(C) 5 (F) There is not enough information to determine a

PTS 4. [5 pts] Given the values of $f(x)$ and $f'(x)$ in the table below, and given that

$$g(x) = \frac{f(2x)}{f(x)},$$

find $g'(1)$.

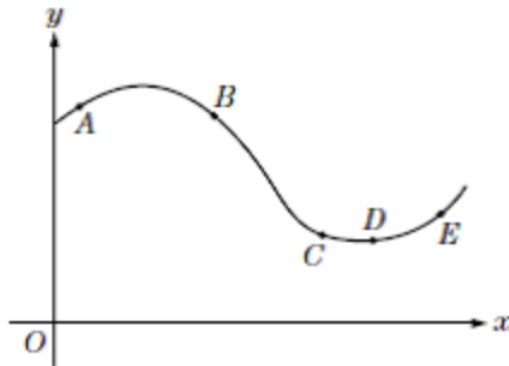| x	f(x)	f'(x)
1	-1	½
2	2	3
3	-3	⅓
4	4	3⁄2

(A) -28 (C) -6 (E) -4

(B) -7 (D) -5 (F) $\frac{3}{2}$

PTS 5. [5 pts] At which point on the graph of $y(x)$, given below, are $y'(x)$ and $y''(x)$ both negative?

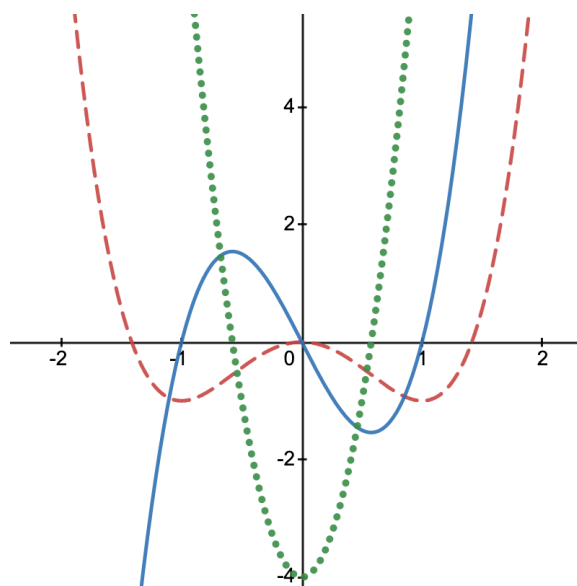(A) A   (C) C   (E) E

(B) B   (D) D

PTS 6. [5 pts] The functions f, f' and f'' are graphed below:

Which of the following statements is true?

(A) f is full, f' is dashed

(B) f is full, f' is dotted

(C) f is dashed, f' is full

(D) f is dashed, f' is dotted

(E) f is dotted, f' is dashed

PTS 7. [5 pts] (Fall 2017) Find ALL the horizontal asymptotes of the function $y = \frac{7|x|-3}{4x-12}$. If there aren't any, write DNE.

(A) DNE

(B) $y = \frac{7}{4}$ and $y = -\frac{7}{4}$

(C) $y = \frac{7}{4}$

(D) $y = -\frac{7}{4}$

(E) $y = 0$

PTS 8. [5 pts] Let $f(x) = 3^{\frac{x^2-9}{x-5}}$. At which value(s) of x does this function have a local maximum?

(A) $x = 0$ (C) $x = 9$ (E) None of these

(B) $x = 1$ (D) $x = 1$ and $x = 9$

9. [5 pts] What is the absolute maximum of the function $f(x) = 5 - 2|x - 1|$ on $[-1, 2]$?

(A) 1   (C) 3   (E) 5

(B) 2   (D) 4

PTS 10. [5 pts] Find the slope of the line tangent to the graph of

$$3x^2 + 2xy + y^2 = 2$$

at $x = 1$.

(A) -2 (C) 2 (E) not defined

(B) 0 (D) 4

PTS 11. [5 pts] What is the absolute maximum of the function

$$f(x) = \begin{cases} -x & \text{if } x \in [-1, 0], \\ x - x^2/4 & \text{if } x \in (0, 3]? \end{cases}$$

(A) 0 (C) $\frac{3}{2}$ (E) $\frac{5}{2}$

(B) 1 (D) 2

12. [5 pts] Compute $\displaystyle\int_0^8 3x^2 - 4x + \frac{x^{\frac{1}{3}}}{6}\, dx$.
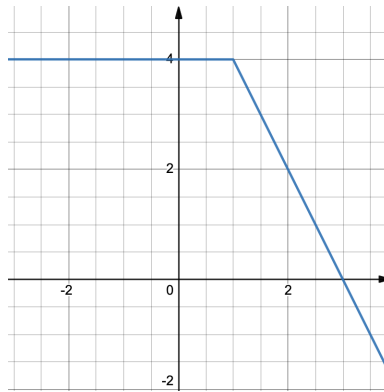
(A) 324   (C) 400   (E) 898

(B) 386   (D) 450

13. [5 pts] Compute $\displaystyle\int_{-1}^2 2f(x) + 1\, dx$ where the graph of f is given below.

(A) 11   (C) 13   (E) 23

(B) 12   (D) 22

14. [5 pts] Using the properties of the definite integral find the value of $\int_1^5 f(x)\,dx$ if it is known that $\int_1^6 f(x)\,dx = 4$, $\int_5^6 f(x)\,dx = 1$.

(A) 1   (C) 3   (E) 5

(B) 2   (D) 4

15. [5 pts] Compute $\int (x+2)\sqrt{x}\,dx$

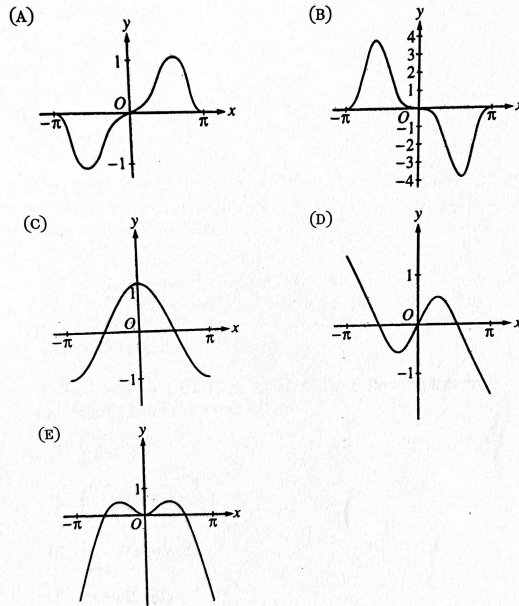(A) $\frac{3}{2}\sqrt{x} + \frac{2}{\sqrt{x}} + C$ (C) $\frac{1}{2}x^2 + 2x + C$ (E) $\frac{1}{2}x^2 - x^{\frac{3}{2}} + x + C$

(B) $\frac{2}{3}x^{\frac{3}{2}} - \sqrt{x} + C$ (D) $\frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{3/2} + C$

_____PTS 16. [5 pts] Which function(s), whose graph is below, has a non-zero average value over the closed interval $[-\pi, \pi]$. *[Recall: The average value of a function f on $[a, b]$ is $f_{ave} = \frac{1}{b-a} \int_a^b f(x)\, dx.]*

(A) A

(B) B

(C) C

(D) D

(E) E

Part II: Free Response

Show all your work neatly and in a structured way.

___PTS 17. [10 pts] (Spring 2022) Use logarithmic differentiation to find the derivative of the function:

$$y = x^{\cos x}.$$

The final answer must be a function of x. Remember to use correct notation to write your final answer.

___PTS 18. [10 pts] (Fall 2022) Determine $\frac{dy}{dx}$ given that $y^3 - 4y = x^2 e^y$. Express $\frac{dy}{dx}$ as a function of x and y.

_____PTS 19. [10 pts] Compute the following limits:

(a) (Spring 2019) $\lim_{x \to 0} \dfrac{\sin x - x}{1 - \cos x}$

(b) (Spring 2023) $\lim_{x \to 0} \dfrac{e^{2x} - 2}{e^x + 1}$

_____PTS 20. [10 pts] Use a linear approximation (or differentials) to estimate $e^{-0.02}$.

_____PTS 21. [10 pts] The table below shows the speed $v(t)$ of a runner at a given time t. Time is given in seconds and speed in meters per second.

| t [s]	v(t) [m/s]
0	0
0.5	4.58
1	6.81
1.5	7.90
2	8.31
2.5	9.22
3	10.14

(a) Using correct units, interpret the meaning of $\int_{0.5}^{2} v(t)dt$.

(b) Use a left Riemann sum with the three subintervals $[0.5, 1]$, $[1, 1.5]$, and $[1.5, 2]$ to approximate $\int_{0.5}^{2} v(t)dt$.

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_____PTS 22. [10 pts] Given $f(x) = \dfrac{e^{x^2+1}}{x \arctan(x)}$, what is $f'(x)$?

_____PTS 23. [10 pts] The velocity of a particle traveling upwards is given by $v(t) = e^{2t}$ feet/second. If the height of the particle is 3 feet at $t = 1$ second, at what time is the height 6 feet?

24. [10 pts] The graph of $y = f'(x)$, the derivative function of $f(x)$, is shown below. Assume that $f(x)$ is defined and continuous on $[-2, 2]$. Give a complete answer to the following questions.
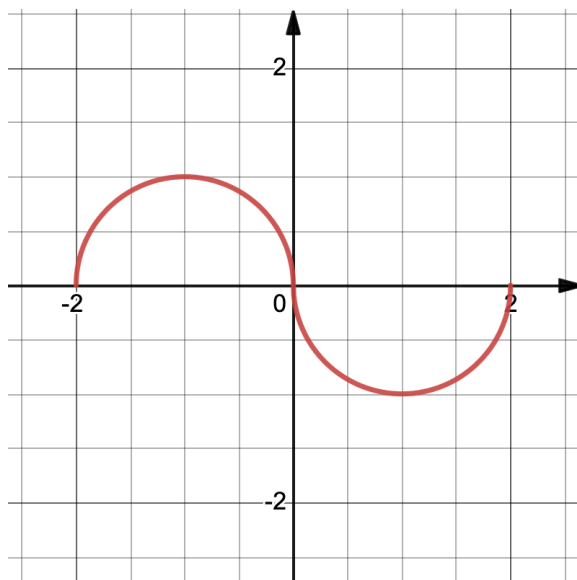
(a) (2pts) What are the x coordinates of all critical points of $f(x)$?
  – If there aren't any, write NONE.

ANSWER:_____

(b) (2pts) What are the x coordinates of all local minima of $f(x)$?
  – If there aren't any, write NONE.

ANSWER:_____

(c) (2pts) What are the intervals where $f(x)$ is concave up?
  – If there aren't any, write NONE.

ANSWER:_____

(d) (2pts) What are the x coordinates of all inflection points of $f(x)$?
  – If there aren't any, write NONE.

ANSWER:_____

(e) (2pts) What are the intervals where $f(x)$ is increasing?
  – If there aren't any, write NONE.

ANSWER:_____

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_PTS 25. [10 pts] Compute $\displaystyle\int \frac{\pi^2 \sin^2 \pi x}{\sec \pi x}\, dx.$

_PTS 26. [10 pts] What is the absolute maximum and the absolute minimum of the function $f(x) = x^4 - 4\pi^3 x + 1$ on the interval $[-1, 4]$?

PTS 27. [10 pts] Find the derivative of the function:

$$f(x) = \frac{x(x-5)^{x-2}}{\sin^5(x)}$$

The final answer must be a function of x. Remember to use correct notation to write your final answer.

PTS 28. [10 pts] What is the function $\phi'(x)$, if $\phi(x) = \int_{x+1}^{x^2} \frac{\sin t}{t^2 + 1}\, dt$?

PTS 29. [10 pts] A plane flies horizontally at an altitude of 2 km and passes directly over a radar located on the ground. When the angle of elevation is 60°, this angle is decreasing at a rate of 30°/min. How fast is the plane at that time?

PTS 30. [10 pts] Dawg Food Inc. wishes to design a can of a volume of $100\ cm^3$ using the least amount of metal as possible. The can has to be cylindrical with a lid and a bottom. Find the dimensions it should use.

(a) (2pts) Sketch the can, label the height with h and the radius with R.

(b) (4pts) Express the exterior area $A(h)$ (in cm^2) of the can in terms of h, **and** give a reasonable domain for A.

(c) (4pts) Find the value of h for which the can requires a minimal amount of metal.

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