TOPOLOGY PRELIM
April 1, 1996

I. Show that $S^2 \times \mathbb{P}^3$ and $S^3 \times \mathbb{P}^2$ have the same universal cover and the same fundamental group, but that they are not homotopically equivalent.

II. State and prove the Tietze extension theorem.

III. Give an example of: (Of course you are to prove the example has the relevant properties)

1) A finite complex X with $\pi_1 X = \mathbb{Z}$, $H_2(X) = H_3(X) = \mathbb{Z}$, $H_n(X) = 0$ for $n > 3$.

2) A contractible subset of $\mathbb{R}^2$ that is not a retract of $\mathbb{R}^2$.

IV. State The Mayer-Victoris exact sequence theorem, and Van-Kampens theorem. Outline the proof of <u>one</u> of these.

V. Let $\mathbb{R}P^3$ be the quotient of S^3 by the antipodal map. Give a cell decomposition of $\mathbb{R}P^3$ and compute its fundamental group and its homology groups. Is $\mathbb{R}P^3$ orientable?

VI. Let $f(x) = \sin\frac{1}{x}$ for $x > 0$, and let S be the union of the graph of f and the segment $\{0, y)| -1 \leq y \leq 1\}$.

Prove that S is connected, but not path connected.

VII. Prove: If A is a compact subset of the metric space X and $\mathcal{U}$ in an open covering of A, then there is a $\delta > 0$ such that if $p \in A$, $N_\delta(p) \subset U$ for some $U \in \mathcal{U}$.