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Team Round / 45 min / 150 points

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 50 points, for a total of 150 points.

For problem 3, the answer should be an exact expression, such as $\pi/2$, $\sqrt{3} + 1$, 8/3, etc. No approximate answers will be accepted.

Problem 1. (Five secret numbers) Suppose there are 5 numbers whose pairwise sums are

5, 9, 20, 24, 31, 35, 39, 42, 46, 61

What are the original 5 numbers? Write them in increasing order.

Problem 2. (The last man standing) n people stand in a circle. Then, every second person is excluded until only one is left. For example, with 10 people, the order of exclusion is as follows:

so the last remaining person is number 5.

Now start with 2005 people. Who will be the last person standing?

Problem 3. (Two triangles) In a triangle ABC, vertices are connected to the points A', B', C' which divide the corresponding sides with the ratio 2 to 1, as in the picture, to form a small triangle KLM in the center. What is the ratio of area of ABC to the area of KLM? (The answer must be greater than 1.)



Authors. Written by Valery Alexeev and Boris Alexeev ©2005.