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\text { Team Round / } 45 \text { min / } 150 \text { points }
$$

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 50 points, for a total of 150 points.

For problem 3, the answer should be an exact expression, such as $\pi / 2$, $\sqrt{3}+1,8 / 3$, etc. No approximate answers will be accepted.

Problem 1. (Five secret numbers) Suppose there are 5 numbers whose pairwise sums are

$$
5,9,20,24,31,35,39,42,46,61
$$

What are the original 5 numbers? Write them in increasing order.

Problem 2. (The last man standing) $n$ people stand in a circle. Then, every second person is excluded until only one is left. For example, with 10 people, the order of exclusion is as follows:

$$
2,4,6,8,10,3,7,1,9
$$

so the last remaining person is number 5 .
Now start with 2005 people. Who will be the last person standing?

Problem 3. (Two triangles) In a triangle $A B C$, vertices are connected to the points $A^{\prime}, B^{\prime}, C^{\prime}$ which divide the corresponding sides with the ratio 2 to 1 , as in the picture, to form a small triangle $K L M$ in the center. What is the ratio of area of $A B C$ to the area of $K L M$ ? (The answer must be greater than 1.)


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