Exam Cover Page

UNIVERSITY OF GEORGIA

DEPARTMENT OF MATHEMATICS
MATH 2250 - FINAL EXAM
SPRING 2024

PRINTED NAME : _____
STUDENT ID : _____
DATE : ____/____/____

GRADE

INSTRUCTOR : _____
CLASS TIME : _____

INSTRUCTIONS

- The exam lasts 3 hours and it has two parts: the first one consists of Multiple Choice (MC) questions, and the second part of Free Response (FR) ones. You must show work for both parts. An unjustified answer will receive no credit. If you are using a shortcut, explain it.
- Your work must be neat and organized. Circle the answer for MC questions and put a box around the final answer for the FR questions. There is only one correct answer for each MC question.
- Smart devices, including smart watches and cell phones, are prohibited and must not be within reach.
- If you plan to use a calculator, only TI-30XS MultiView (the name must match exactly) is permitted; no other calculators or sharing of calculators is allowed.
- Provide an exact answer for each problem. Answers containing symbolic expressions such as cos(3) and ln(2) are perfectly acceptable.
- If additional space is needed, use the last two pages. Write 'ctd' (continued) in the designated area and continue on the scrap paper by first writing the problem number and then continuing your solution. Work outside the specified area without any indication, will not be graded.

Information

Date : Thursday, May 2nd 2024
Time : 7 pm to 10 pm
Venue : See instructor, not your regular classroom!

Calculator : TI 30XS MultiView exclusively

Bring your UGA ID

What you should know

Definitions and Theorems to State:
-The (limit) definition of the derivative of $f(x)$
-The definition of continuity at $x = a$

Properties to Cite when Using:
-The relationship of continuity to differentiability
-The Extreme Value Theorem (Closed Interval Method)
-First derivative test for local extrema
-Second derivative test for local extrema
-Second derivative criterion for concavity
-L'Hopital's rule
-Fundamental Theorems of Calculus (Part 1 and Part 2)

Properties you will be responsible for:
-Properties of logarithmic and exponential functions
-Other precalculus-level formulas (in addition to those listed below)

Limits:
-Be able to find the limits and one-sided limits of functions (even if not continuous), both analytically and graphically
-Find limits that approach infinity or have an infinite limit
-Determine horizontal asymptotes and vertical asymptotes of a function; justify your answer using one or more limits
-Be able to use L'Hopital's Rule to find limits (and identify and state the appropriate indeterminate forms that allow you to do so)

-Verify continuity (analytically and graphically)
-Determine intervals on which a function is continuous
-Be able to "repair" a removable discontinuity by (re)defining the function at that x-value
-Determine the value of a parameter that makes a piecewise function continuous where the two pieces meet

Derivatives:
-Be able to find the derivative $f'(x)$ from the limit definition of the derivative
-Be able to use rules to find the derivative; know all rules from back of book through inverse trig function (no hyperbolic or parametric, no $\text{arcsec}(x)$, $\text{arccot}(x)$, or $\text{arccsc}(x)$)
-Implicit differentiation
-Be able to compute derivatives at specific points using limited information (e.g. a table)
-Be able to find an equation of the tangent line at a point
-Be able to understand/interpret the slope of a function
-Logarithmic differentiation

Proof-based Problems:
-Use differentiation of the appropriate inverse function to verify the differentiation rule for $\ln(x)$
-Use differentiation of the appropriate inverse function to verify the differentiation rule for $\arcsin(x)$, $\arccos(x)$, and $\arctan(x)$ (including an appropriate right triangle diagram or a Pythagorean identity)

Applications of Derivatives
-Applications involving a tangent line
-Be able to find and use the linearization
-Be able to find and use the differential
-Position, displacement, velocity, acceleration problems
-Interpret the derivative as a rate of change in a wide range of contexts
-Related rates
-Understand the relationship between (first and second) derivatives and curve behavior; curve sketching from derivative information
-Determine all extrema of a function on a closed interval
-Applied optimization (open and/or closed interval); justify that you have a max or min

Integration
-Antiderivatives: find the most general antiderivative and solve initial value problems
-Understand the definite integral as net area
-Apply properties of the definite integral
-Be able to use the definite integral to compute and interpret
- net area
- total area
- area between two curves

-Estimate a definite integral using well-chosen sums with a small number of rectangles (left, right, midpoint), and interpret your answer

-Use summation notation to express a Riemann sum with N rectangles of equal width and right endpoints, using only the summation symbol, k, N, and numbers
-Compute a definite integral:

- by interpreting it as area
- by Evaluation Theorem (FTC 2)
- by integration via substitution

Terminology to be familiar with (in addition to terminology listed in sections above):
-average rate of change/secant slope, average velocity
-instantaneous rate of change/tangent slope
-tangent lines and linearization of a function at a point
-domain
-critical points (critical numbers), inflection points
-increasing, decreasing, concave up, concave down
-local (relative) extrema
-absolute (global) extrema

Penalties (approximately 20% of problem's points value for each issue):
-Improper use of $+C$ or missing $+C$
-Improper use of limit notation
-Improper use of integral or sigma
-Improper use of "=" (like $y = x^3 = 3x^2$)
-Improper algebraic notation (missing parentheses, incorrect variable name, etc.)

Remarks for students:
-Problems may combine multiple topics/techniques.
-You do not have to simplify your answers.
-Calculator **TI-30XS Multiview only**! No other calculators are allowed, and sharing of calculators is not allowed.
-You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem. -You will leave your backpacks at the front of the room; a backpack that rings or buzzes will be taken out to the hallway and left there.
-No smart watches are allowed during the exam; smart devices (including smart watches and cell phones) may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.

Formulas to Remember

- Distance between (x_1, y_1) and (x_2, y_2): $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Triangles and Trig
  - perimeter (add up side lengths)
  - area: $A = \frac{1}{2}bh$
  - Be able to use properties of similar triangles.
  - Pythagorean Theorem for right triangles: $a^2 + b^2 = c^2$
  - right triangles and acute angle trig: SOH-CAH-TOA
$\tan(x) = \dfrac{\sin(x)}{\cos(x)}$	$\cot(x) = \dfrac{\cos(x)}{\sin(x)}$
$\csc(x) = \dfrac{1}{\sin(x)}$	$\sec(x) = \dfrac{1}{\cos(x)}$
  - $\sin^2(x) + \cos^2(x) = 1$
  - Your trig differentiation formulas assume that your angle is in radians. (Why?)
- Circles
  - area: $A = \pi r^2$
  - circumference: $C = 2\pi r$
  - Equation of the circle of radius r centered at (h, k): $(x - h)^2 + (y - k)^2 = r^2$
- Rectangles
  - area: $A = lw$
  - perimeter: $P = 2l + 2w$
- Cylinder
  - volume: $V = \pi r^2 h$
  - surface area: $S = 2\pi r^2 + 2\pi rh$ (includes base and lid)
- Rectangular prisms
  - volume: $V = lwh$
  - surface area: $S = 2lw + 2wh + 2lh$ (includes top and base)
- Circular cone
  - volume: $V = \frac{1}{3}\pi r^2 h$
- Sphere
  - volume: $V = \frac{4}{3}\pi r^3$