By providing my signature below, I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (print): Solutions Name (sign): ______

Student ID (81#): ______

Instructor's Name: ______ Class Time: ______

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion, then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Cell phones and smart watches are NOT allowed; smart devices (including smart watches and cell phones) may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.
- You are only allowed to use a **TI-30XS Multiview** calculator; the name must match exactly. No other calculators are permitted, and sharing of calculators is not permitted.
- You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem.

Problem Number	Points Possible	Points Earned
1	26	
2	10	
3	8	
4	10	
5	18	
6	18	
7	11	
8	26	
9	14	
10	10	
11	15	
Total:	166	

1. Determine the following limits; briefly explain your thinking on each one. If you apply L'Hopital's Rule, indicate where you have applied it and why you can apply it. If your final answer is "does not exist," ∞, or, $-\infty$, briefly explain your answer. (You will not receive full credit for a "does not exist" answer if the answer is ∞ or $-\infty$.)

(a) [6 pts] $\lim_{x\to 2} \sqrt[3]{\dfrac{x^2+x-6}{x^2-x-2}} = \lim_{x\to 2} \sqrt[3]{\dfrac{(x+3)(x-2)}{(x+1)(x-2)}} = \lim_{x\to 2} \sqrt[3]{\dfrac{x+3}{x+1}}$

$= \sqrt[3]{\dfrac{2+3}{2+1}} = \sqrt[3]{\dfrac{5}{3}}$

(b) [6 pts] $\lim_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7} =$

$= \lim_{x\to 7} \dfrac{\sqrt{x+2}-3}{x-7} \cdot \dfrac{\sqrt{x+2}+3}{\sqrt{x+2}+3}$

$= \lim_{x\to 7} \dfrac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$

$= \lim_{x\to 7} \dfrac{x-7}{(x-7)(\sqrt{x+2}+3)}$

$= \lim_{x\to 7} \dfrac{1}{\sqrt{x+2}+3}$

$= \dfrac{1}{\sqrt{7+2}+3} = \dfrac{1}{6}$

or

IF $\frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 7} \frac{1}{2}(x+2)^{-1/2}$

$= \lim_{x\to 7} \dfrac{1}{2\sqrt{x+2}}$

$= \dfrac{1}{2\sqrt{7+2}}$

$= \dfrac{1}{6}$

(c) [7 pts] $\lim_{x\to 0} \frac{1-\cos(x)}{x^2} =$

IF $\frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{\sin(x)}{2x} \to \frac{0}{0}$

$\overset{LH}{=} \lim_{x\to 0} \frac{\cos(x)}{2}$

$= \frac{1}{2}$

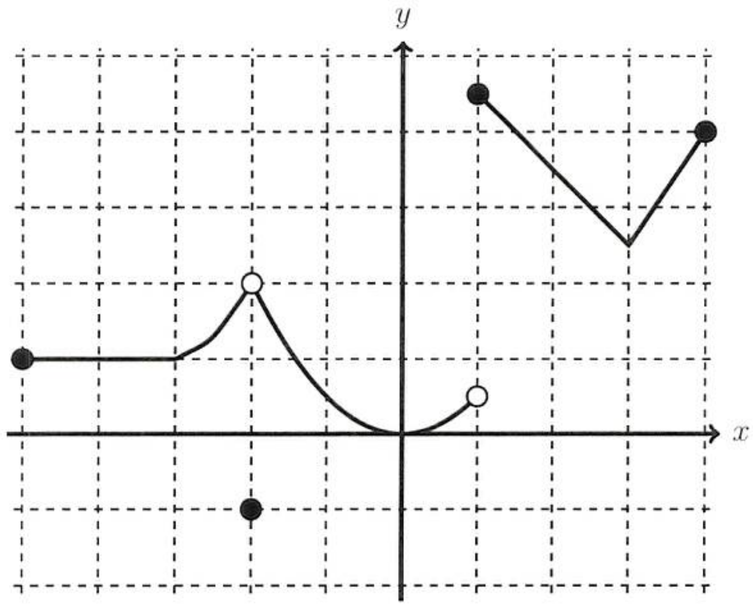
(d) [7 pts] $\lim_{x\to\infty} xe^{-3x} =$

$= \lim_{x\to\infty} \frac{x}{e^{3x}} \longrightarrow \frac{\infty}{\infty}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{1}{3e^{3x}}$ (as $x\to\infty$, $e^{3x}\to\infty$)

$= 0$

2. Consider the graph of $y = f(x)$ defined on the closed interval $[-5, 4]$ given below. The grid lines are one unit apart. Based on the graph, answer the following questions.



______ (a) [3 pts] Determine all values of a such that $f(a)$ exists and $\lim_{x \to a} f(x)$ does not exist.

$a = 1$

______ (b) [3 pts] Determine all values of x in the interval $(-5, 4)$ such that $f(x)$ exists and $f'(x)$ does not exist.

$x = -2, 1, 3, -3$

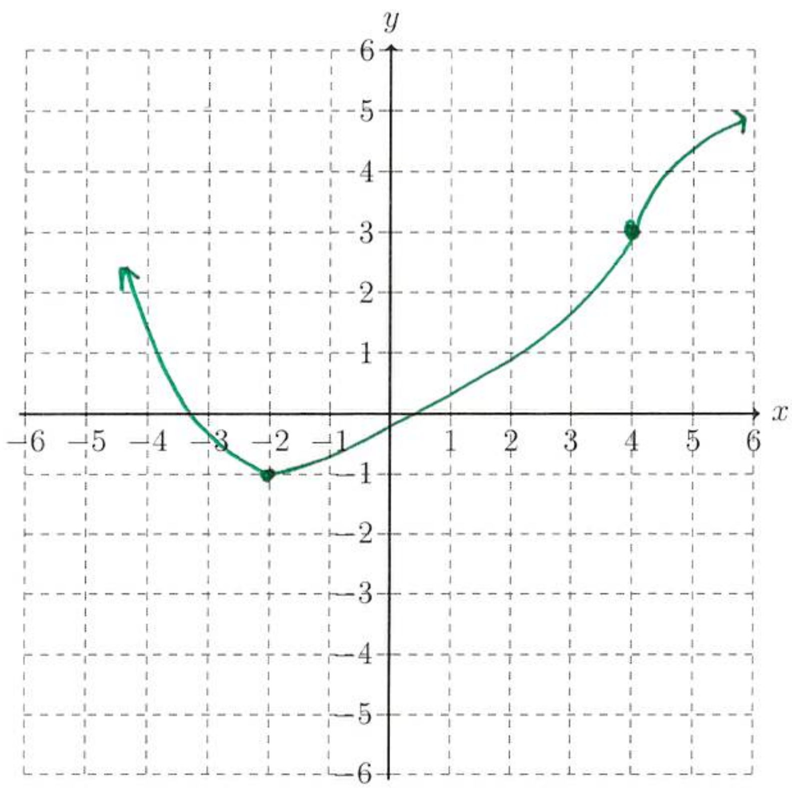
↑ if it looks like a sharp corner

______ (c) [4 pts] Determine all values of x in the interval $(-5, 4)$ such that $f'(x) < 0$.

$(-2, 0), (1, 3)$

x	**Derivatives**
$x < -2$	$f'(x) < 0, f''(x) > 0$
$-2 < x < 4$	$f'(x) > 0, f''(x) > 0$
$x > 4$	$f'(x) > 0, f''(x) < 0$

3. [8 pts] Suppose a twice-differentiable function $f(x)$ satisfies the properties given above and $f(x)$ passes through the points $(-2, -1)$ and $(4, 3)$. Then sketch the graph of the function f below.



4. [10 pts] Use the **limit definition of the derivative** to determine the derivative of

$$f(x) = \frac{1}{3x+7}.$$

No points will be awarded for the application of differentiation rules (and L'Hopital's Rule is not allowed.)
Show all steps.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3(x+h)+7} - \frac{1}{3x+7}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{3x+3h+7} - \frac{1}{3x+7}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3x+7-(3x+3h+7)}{(3x+3h+7)(3x+7)}}{h}$$

$$= \lim_{h \to 0} \frac{3x+7-3x-3h-7}{h(3x+3h+7)(3x+7)}$$

$$= \lim_{h \to 0} \frac{-3h}{h(3x+3h+7)(3x+7)}$$

$$= \lim_{h \to 0} \frac{-3}{(3x+3h+7)(3x+7)}$$

$$= \frac{-3}{(3x+7)^2}$$

5. Determine the first derivative of each of the following functions. Remember to use correct notation to write your final answer.

(a) [6 pts] $f(t) = \dfrac{4t^5 - 3}{7t} + e^{9t}$

$$f'(t) = \frac{7t(20t^4) - (4t^5-3)(7)}{(7t)^2} + 9e^{9t}$$

(b) [6 pts] $h(x) = \cos(x)\arcsin(x)$

$$h'(x) = \cos(x)\left(\frac{1}{\sqrt{1-x^2}}\right) + \arcsin(x)(-\sin(x))$$

(c) [6 pts] $k(y) = \tan(9y) - \ln(5y^3 - y^2 + 4)$

$\cancel{k'(y) = 9\sec^2(9y)}$

$$k'(y) = 9\sec^2(9y) - \frac{15y^2 - 2y}{5y^3 - y^2 + 4}$$

6. Determine $\frac{dy}{dx}$.

(a) [8 pts] $y = x^{\sin(x)}$

$$\ln(y) = \ln(x^{\sin(x)})$$

$$\ln(y) = \sin(x)\ln(x)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(\sin(x)\ln(x))$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{\sin(x)}{x} + \ln(x)\cos(x)$$

$$\frac{dy}{dx} = y\left(\frac{\sin(x)}{x} + \ln(x)\cos(x)\right)$$

$$\frac{dy}{dx} = x^{\sin(x)}\left(\frac{\sin(x)}{x} + \ln(x)\cos(x)\right)$$

(b) [10 pts] $y^3 - 4y = x^2e^y$

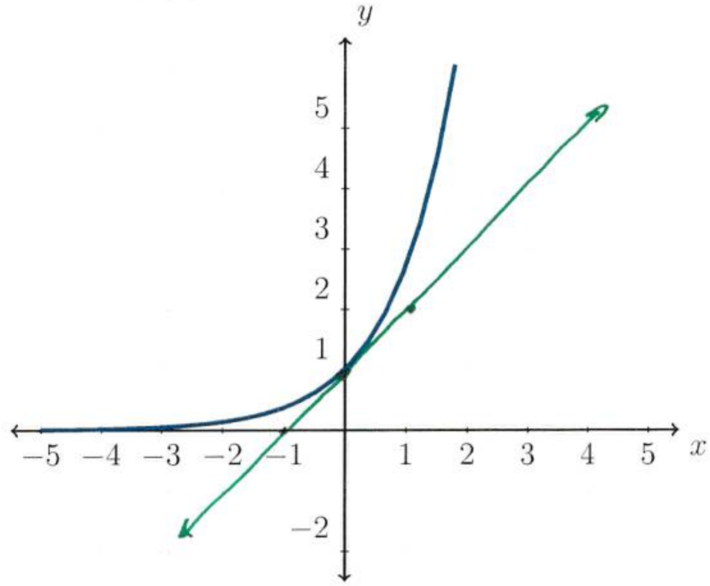
$$3y^2\frac{dy}{dx} - 4\frac{dy}{dx} = x^2e^y\frac{dy}{dx} + e^y(2x)$$

$$3y^2\frac{dy}{dx} - 4\frac{dy}{dx} - x^2e^y\frac{dy}{dx} = 2xe^y$$

$$\frac{dy}{dx}(3y^2 - 4 - x^2e^y) = 2xe^y$$

$$\frac{dy}{dx} = \frac{2xe^y}{3y^2-4-x^2e^y}$$

7. Consider $f(x) = e^x$. It has been graphed below.



(a) [7 pts] Write the equation of the tangent line of $y = f(x)$ at the point $(0, 1)$ and then **graph the line** on the grid provided above.

Slope: $f'(x) = e^x$

$f'(0) = e^0 = 1$

Eqn: $y - 1 = 1(x - 0)$

$y = x + 1$

(b) [4 pts] Use the linearization of $y = f(x)$ at $x = 0$ to approximate $e^{0.05}$. (You don't need to simplify.) Use the graph to explain whether or not your approximation is an overestimate or underestimate of $e^{0.05}$.

$L(x) = x + 1$

$L(0.05) = 0.05 + 1 = 1.05$

$\Rightarrow e^{0.05} \approx 1.05$

This is an underestimate b/c the tangent line is below the graph of f(x)

8. Evaluate the following. You do not need to simplify your answers.

_______ (a) [8 pts] $\int \left(\frac{2}{x^3} + \frac{8}{x} + \frac{1}{1+x^2} \right) dx$

$$= \frac{-2x^{-2}}{2} + 8\ln|x| + \arctan(x) + C$$

_______ (b) [6 pts] $\int \frac{\sqrt{\ln(x)+4}}{x} dx$

$$= \int \sqrt{u}\, du = \int u^{1/2} du$$

$u = \ln(x) + 4$

$du = \frac{1}{x} dx$

$$= \frac{u^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{2}{3}(\ln(x)+4)^{3/2} + C$$

(c) [6 pts] $\int_{-2}^{1}(x^2+x+1)dx$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} + x \right]_{-2}^{1}$$

$$= \frac{1^3}{3} + \frac{1^2}{2} + 1 - \left(\frac{(-2)^3}{3} + \frac{(-2)^2}{2} - 2 \right)$$

(d) [6 pts] $\frac{d}{dx}\int_{2}^{x}\left(\csc^2(t)+2^t\right)dt$

$$= \csc^2(x) + 2^x$$

9. Suppose that from a height of 6 feet above the ground, a ball is tossed vertically in such a way that its velocity function is given by $v(t) = 32 - 32t$, where t is measured in seconds and v in feet per second. Assume that this function is valid for $t \geq 0$.

   (a) [4 pts] After how many seconds will the ball change direction and begin to fall to the ground?

$$v(t) = 0$$
$$32 - 32t = 0$$
$$32 = 32t$$
$$t = 1 \text{ second}$$

The ball will begin to fall after 1 second.

   (b) [6 pts] Determine the distance between the ball and the ground after 1.5 seconds.

$$s(t) = \int (32 - 32t)\,dt$$
$$= 32t - \frac{32t^2}{2} + C$$

$s(0) = 6$:

$$32(0) - 16(0)^2 + C = 6$$
$$C = 6$$
$$s(t) = 32t - 16t^2 + 6$$
$$s(1.5) = 32(1.5) - 16(1.5)^2 + 6$$
$$= 18 \text{ ft}$$

The distance between the ball and the ground after 1.5 seconds is 18ft.

   (c) [4 pts] After how many seconds will the ball hit the ground?

$$s(t) = 0$$
$$-16t^2 + 32t + 6 = 0$$
$$t = \frac{-32 \pm \sqrt{32^2 - 4(-16)(6)}}{2(-16)}$$
$$= 1 + \frac{\sqrt{1408}}{32},\quad 1 - \frac{\sqrt{1408}}{32}$$

The ball will hit the ground after $1 + \frac{\sqrt{1408}}{32}$ seconds

10. [10 pts] Air is leaking from a spherical balloon at a rate of 5 cm^3/min. Determine the rate at which the radius of the balloon is changing when the radius of the balloon is 10 cm. (You do not need to simplify your answer; decimal approximations will not be accepted.)
<u>Hint:</u> The volume of a sphere is $V = \frac{4}{3}\pi r^3$.

$$\frac{dV}{dt} = -5 \text{ cm}^3/\text{m}$$

$$r = 10$$

$$\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

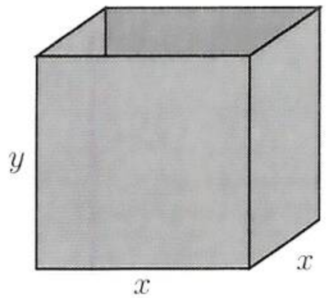
$$-5 = 4\pi(10^2)\frac{dr}{dt}$$

$$\frac{-5}{400\pi} \text{ cm/min} = \frac{dr}{dt}$$

The radius of the balloon is <u>decreasing</u> at $\frac{5}{400\pi}$ cm/min when the radius of the balloon is 10 cm.

11. A rectangular box with a square base and an open top is to be constructed.

_______ (a) [5 pts] Suppose the volume of the box is 200 cubic inches. Write a formula for the surface area of the box as a function of x, the lengths of its base, and determine the domain of the function.



$V = x^2 y$

$200 = x^2 y$

$\frac{200}{x^2} = y$

$SA = 4xy + x^2$

$= 4x\left(\frac{200}{x^2}\right) + x^2$

$= \frac{800}{x} + x^2$

domain: $(0, \infty)$
b/c $x > 0$ and
$y = \frac{200}{x^2} > 0$

_______ (b) [10 pts] The manufacturing company has decided that the box will need to have a volume of 216 cubic inches. The material chosen to construct the box will cost \$0.20/square inch for the base and \$0.30/square inch for the sides. Then the cost (in dollars) of the box is given by

$$C(x) = \frac{259.2}{x} + 0.2x^2$$

where x is represents the lengths of the base of the box. What should the dimensions of the box be to minimize the cost of making the box?

$C(x) = 259.2x^{-1} + 0.2x^2$

$C'(x) = -\frac{259.2}{x^2} + 0.4x$

$= \frac{-259.2 + 0.4x^3}{x^2}$

$C'(x) = 0$:

$-259.2 + 0.4x^3 = 0$

$0.4x^3 = 259.2$

$x^3 = \frac{259.2}{0.4}$

$x = \sqrt[3]{\frac{259.2}{0.4}}$

f': $\xleftarrow{\quad - \quad} \sqrt[3]{\frac{259.2}{0.4}} \xrightarrow{\quad + \quad}$ ← First derivative test

$y = \frac{216}{x^2} = \frac{216}{\left(\sqrt[3]{\frac{259.2}{0.4}}\right)^2}$

A box of volume 216 in^3 should have dimensions $\sqrt[3]{\frac{259.2}{0.4}}$ in $\times$ $\sqrt[3]{\frac{259.2}{0.4}}$ in $\times$ $\frac{216}{\left(\sqrt[3]{\frac{259.2}{0.4}}\right)^2}$ in to minize the cost of making it.

This page is extra space for work. **Do not detach this page.** If you want us to consider the work on this page, you should print your name, instructor and class meeting time below. For the problems where you want us to look at this work, please write "see last page" next to your work on the problem page so that we know to look here.

Name (print): ______________________ Instructor (print): ______________________

Class Meeting Time: ______________