By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (sign): _____ Name (print): _____

Student Number: _____

Instructor's Name: _____ Class Time: _____

Problem Number	Points Possible	Points Made
1	18	
2	26	
3	16	
4	20	
5	22	
6	20	
7	15	
8	20	
9	15	
10	15	
11	15	
12	15	
13	30	
14	10	
15	15	
Total:	272	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- You are only allowed to use a **TI-30XS Multiview** calculator. No other calculators are permitted, and sharing of calculators is not permitted.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

1. Determine the following limits. If you answer with ∞ or $-\infty$, briefly explain your thinking. Print your final answer in the box provided.

_____ (a) [5 pts] $\lim_{x\to 2}(3x^2+7x-5)$

answer:

_____ (b) [5 pts] $\lim_{x\to 1^-}\dfrac{2x}{x-1}$

answer:

_____ (c) [8 pts] $\lim_{x\to\infty}\dfrac{\ln(5x)}{x^3+1}$

answer:

2. Determine the first derivative of each of the following functions. Print your answer in the box provided. *You do not have to simplify your answers or explain your steps.*

_____ (a) [4 pts] $f(x) = 8x^3 - 15x + 12$

$f'(x) =$

_____ (b) [6 pts] $g(t) = \dfrac{\sin(t)}{t}$

$g'(t) =$

_____ (c) [6 pts] $f(x) = \dfrac{e^x}{2x+1}$

$f'(x) =$

_____ (d) [10 pts] $h(x) = (4x-3)^2 \arctan(x)$

$h'(x) =$

3. (a) [8 pts] Determine $\dfrac{dy}{dx}$ for the equation $y^3 - x^4 y = 6$. Print your answer in the box provided. You do not have to simplify your answer.

$\dfrac{dy}{dx} =$

(b) [8 pts] Determine an equation of the tangent line to the curve $y^3 - x^4 y = 6$ at the point $(1, 2)$.

Equation:

4. Determine the following indefinite integrals. Print your answer to each part in the box provided.

——— (a) [4 pts] $\int \left(-4x^7 + 8x^5 + 12\right) dx$

Final answer:

——— (b) [6 pts] $\int \left(\sec^2(t) + \frac{1}{t}\right) dt$

Final answer:

——— (c) [10 pts] $\int \frac{x^4}{\sqrt{x^5+3}} dx$

Final answer:

5. Evaluate the following definite integrals. Print your answer in the box provided.

_____ (a) [6 pts] $\displaystyle\int_1^8 \left(x^{2/3} - \frac{1}{x^{4/3}}\right)\, dx$

Value:

_____ (b) [6 pts] $\displaystyle\int_0^{1/2} \frac{-1}{\sqrt{1-x^2}}\, dx$

Value:

_____ (c) [10 pts] $\displaystyle\int_0^{\pi/4} \sin(4x)e^{\cos(4x)}\, dx$

Value:

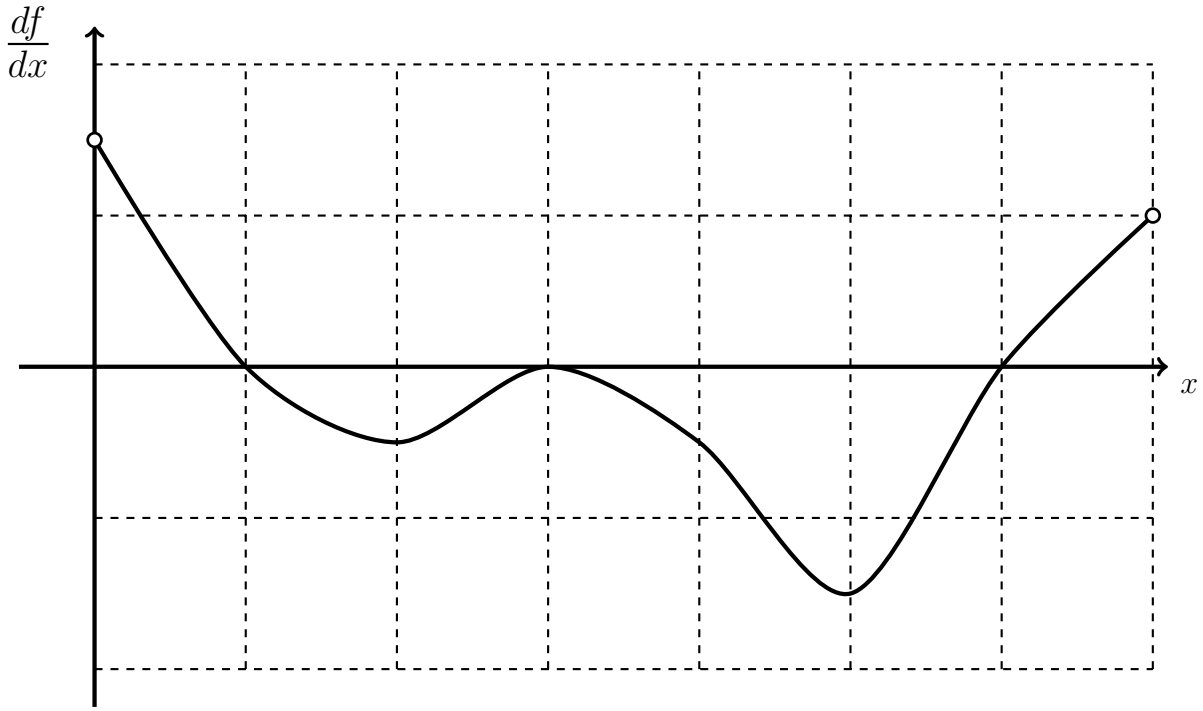
6. (a) [5 pts] State the limit definition of the derivative of $f(x)$.

(b) [10 pts] Use the limit definition of the derivative to show that the derivative of $f(x) = 12x - 2x^2$ is $f'(x) = 12 - 4x$. (You will receive 0 points for using the power rule.)

(c) [5 pts] Determine all values of x for which the graph of $f(x) = 12x - 2x^2$ has a horizontal tangent line.

7. [15 pts] Determine the absolute maximum and absolute minimum values of $f(x) = 2x\sqrt{9-x}$ on the interval $[-1, 9]$.

8. The graph below is the graph of **the derivative of** $f(x)$. Use it to answer the questions that follow. The grid lines are one unit apart, and the domain of f is $(0, 7)$.



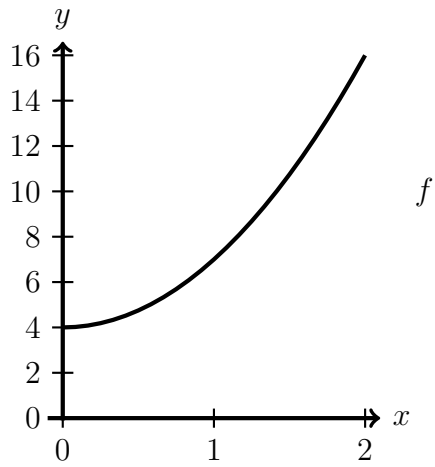
(a) [5 pts] Determine all critical numbers (critical points) of f.

(b) [5 pts] Determine the intervals on which f is increasing.

(c) [5 pts] Determine all values of x at which f has a local minimum.

(d) [5 pts] Determine the intervals on which f is concave up.

9. For this problem, use $f(x) = 3x^2 + 4$ on the interval $[0, 2]$. Its graph is provided to the right.



$f(x) = 3x^2 + 4$

(a) [5 pts] Determine a Riemann sum for f on the interval $[0, 2]$ using 3 subintervals of equal width and using right endpoints on each subinterval.

(b) [5 pts] Is your Riemann sum above an over- or under-estimate of the integral $\int_0^2 f(x)\,dx$? Explain how you can tell, *without doing any calculations or working out the answers*, whether it's an over-estimate or an under-estimate. (You may want to illustrate the Riemann sum on the graph of f provided above.)

(c) [5 pts] Use summation (sigma) notation to write an expression for a Riemann sum for f on the interval $[0, 2]$ using n subintervals of equal width and using right endpoints on each subinterval. You do not have to work out the value of the sum, but your sum should involve only $\sum_{k=1}^{n}$, the variables k and n, and numbers.

10. Use the values of the given definite integrals to determine the quantities below.

$$\int_1^7 f(x)\,dx = -8, \qquad \int_3^7 f(x)\,dx = 12, \qquad \int_1^7 g(x)\,dx = 9$$

(a) [5 pts] $\int_1^7 (2f(x) - 5g(x))\,dx$

(b) [5 pts] $\int_1^3 f(x)\,dx$

(c) [5 pts] $\int_1^7 (g(t) - t^2)\,dt$

11. The charts below contain information about a function f and its derivative. Assume that f is differentiable on $[-2, 1]$. Use the charts to answer the questions that follow.

x	-2	-1	0	1
$f(x)$	3	2	0	-1

x	-2	-1	0	1
$f'(x)$	$-\frac{1}{8}$	$-\frac{1}{3}$	-1	0

(a) [5 pts] Determine the linearization of f at $x = -1$.

(b) [5 pts] Use your linearization above to estimate the value of $f(-1.5)$.

(c) [5 pts] Suppose you also know that f' is continuous on $[-2, 1]$. Explain why the graph of f must have an inflection point somewhere in the interval $[-2, 1]$.

12. [15 pts] A diesel truck develops an oil leak. The oil drips onto the dry ground in the shape of a circular puddle. Assuming that the leak begins at time $t = 0$ and that the radius of the oil slick increases at a constant rate of .05 meters per minute, determine the rate of change of the area of the puddle 4 minutes after the leak begins.

13. A landscape designer plans to construct a rectangular garden whose area is 2000 square meters. One side will consist of a wrought iron fence which costs \$90 per meter. The remaining three sides will be constructed from chain link fence costing \$25 per meter.

(a) [15 pts] Determine a function for the total cost $C(x)$ of the garden, where x is the length of wrought iron fence used (in meters).

(b) [15 pts] What dimensions of the garden will minimize the total cost? Use calculus techniques to show that the dimensions result in the minimum possible cost.

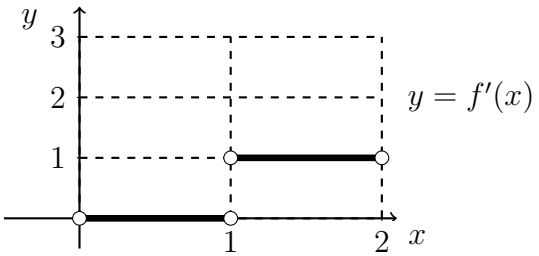
14. [10 pts] Let $y = \ln(x)$. Show that $\dfrac{dy}{dx} = \dfrac{1}{x}$ by solving the equation $y = \ln(x)$ for x and then using implicit differentiation. Your final answer should be $\dfrac{dy}{dx}$, given as a function of x.

15. Information about a function, f, and its derivative is given below. Use the information to answer the questions that follow.

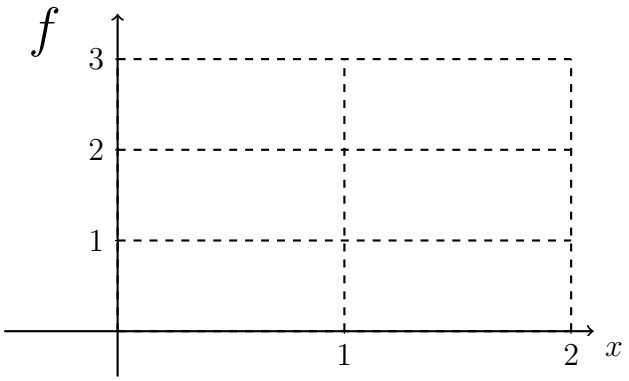
Information about f:

$$\lim_{x\to 0^+} f(x) = 2,$$
$$\lim_{x\to 2^-} f(x) = 1,$$

This is the graph of the **derivative** of f:



(a) [5 pts] Make a rough sketch of the graph of $y = f(x)$. (Hint: Think about slopes.)



(b) [5 pts] Determine $\lim_{x\to 1^-} f(x)$.

(c) [5 pts] Determine $\lim_{x\to 1^+} f(x)$.

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____