

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): \_\_\_\_\_

Name (print): \_\_\_\_\_

Student Number: \_\_\_\_\_

Instructor's Name: \_\_\_\_\_

Class Time: \_\_\_\_\_

Problem Number	Points Possible	Points Made
1	0	
2	22	
3	24	
4	14	
5	10	
6	10	
7	10	
8	10	
Total:	100	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.
- Common identities:

$$\begin{aligned}\cos(\alpha + \beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta), \\ \sin(\alpha + \beta) &= \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).\end{aligned}$$

1. [2 Bonus] Common Knowledge: How many stages will the 2020 Lotto Belgium Tour have?

2. Determine all of the values of  $x$  for each question below that satisfy the given equation.

\_\_\_\_\_ (a) [7 pts]  $\log_9(3x - 1) = \frac{3}{2}$

\_\_\_\_\_ (b) [7 pts]  $\ln(x) - 2 = \ln(3x - 1)$

\_\_\_\_\_ (c) [8 pts]  $13 \cdot 2^{x+1} = 20 \cdot 5^{4x-1}$

3. For each part below a function is given. State whether or not the function gets close to some fixed value as  $x$  gets bigger or if it cannot be bounded above or below as  $x$  gets bigger. Also state if the function is exponential, linear, or something else.

(a) [6 pts]  $u(x) = 5 \cdot 6^x$

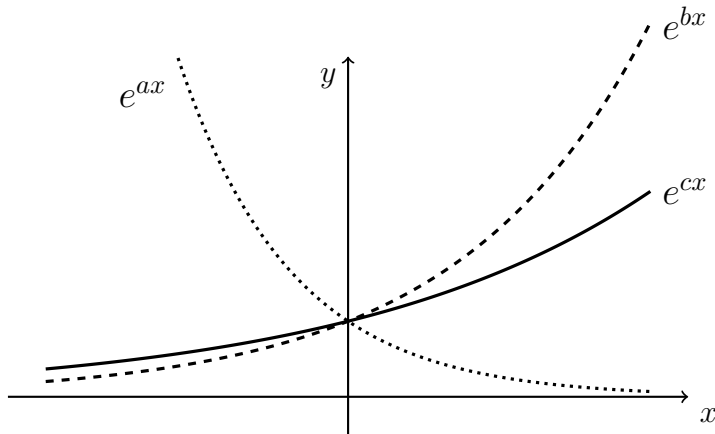
(b) [6 pts]  $v(x) = 4 + 2x$

(c) [6 pts]  $w(x) = 10 \cdot e^{-3x}$

(d) [6 pts]  $s(x) = 5 - x$

4. Compare the growth and decay rates as described in each question below. Briefly justify your conclusions.

- (a) [8 pts] Three exponential functions are given below. Order the values of  $a$ ,  $b$ , and  $c$ . That is rank the numbers in order from smallest to highest. Provide a brief justification for your answer.



- (b) [6 pts] Which function grows faster as  $x$  gets bigger,

$$\begin{aligned}h(x) &= 2.7^x, \\g(x) &= e^x.\end{aligned}$$

(Provide a justification for your answer beyond just testing some numbers for  $x$ .)

5. [10 pts] The domain of the function

$$K(x) = x^2$$

is  $(-\infty, \infty)$ . Determine a subset of the domain where  $K(x)$  is one-to-one when restricted to your new domain. (In other words, find a new domain in the form  $(a, b)$  or  $[a, b)$  where the function will be one-to-one.) Briefly justify why the function is one-to-one on your new, restricted domain. Also, determine the inverse of the function when the domain is restricted.

6. [10 pts] The President of the United States recently urged the Federal Reserve to lower its lending rates so that they will be negative. Your good friend, who is a member of the Board of Governors of the Federal Reserve, asks you for your mathematical advice. Suppose that someone deposits \$1,000 into an account that is compounded monthly. Using the standard formula for compounded interest, determine the interest rate so that the account will have \$800 at the end of two years.

7. The population of *Pseudoeurycea goebeli* in a preserve on the side of the Tajumulco volcano is approximated using a logistic function,

$$P(t) = \frac{C}{1 - \frac{1}{3}e^{-rt}}.$$

The initial population is estimated to be three thousand, and after ten years the population is estimated to be two-thousand and five-hundred.

- (a) [5 pts] Determine the values of  $C$  and  $r$ .
- (b) [5 pts] What is the long-term population? (This is the number the population gets close to after a very long time. Briefly justify your result.)

8. [10 pts] A sample of bacteria from the Oconee River includes about four million individuals. The sample is placed in a new, pristine container, and after seven days it includes five million individuals. Assuming that the population can be approximated using an exponential function determine an approximation for the number of individuals after ten days.



Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): \_\_\_\_\_ Instructor (print): \_\_\_\_\_ Time: \_\_\_\_\_