

Print and sign a brief statement stating this is your work and you used no other resource:

Name (sign): _____

Name (print): _____

Student Number: _____

Instructor's Name: _____

Class Time: _____

Problem Number	Points Possible	Points Made
??	0	
??	27	
??	15	
??	15	
??	16	
??	17	
??	10	
Total:	100	

- If you need extra space use the last page.
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion then state the theorem used by name.
- Your work must be **neat**. This will make it easier to properly grade and give you the credit you deserve.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Please turn off your mobile phone.
- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta), \quad \sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

1. Common Knowledge: How will the win by Magdeleine Vallieres in Rwanda impact her career?

2. Determine all of the values of x for each question below that satisfy the given equation. If no values of x satisfy the equation provide a brief justification as to how you arrived at your conclusion.

(a) $e^{3x+1} = 8$.

(b) $\ln(x+1) = \ln(3x-1) + 5$.

(c) $\log_2(2x + 1) + \log_2(x - 1) = 3.$

(d) $8e^{1-2x} = 4^x.$

3. Use the properties of logarithms to answer each question below.

- (a) Expand the expression below as far as possible so it only contains the logarithms of single terms,

$$\log_7 (7 \cdot x^4 \cdot y^{-3})$$

(Your final result should not have the logarithm of a known constant.)

- (b) Expand the expression below as far as possible so it only contains the logarithms of single terms,

$$\log_{13} \left(x^4 \cdot \frac{w^{1/3} z^3}{\sqrt{y}} \right)$$

(Your final result should not have the logarithm of a known constant.)

- (c) Combine the terms in the following expression into a single logarithm

$$4 \log_3(w) - \frac{1}{2} \log_3(u) + \frac{4}{7} \log_3(x).$$

4. An exponential function is given by

$$p(x) = 5 + Ae^{rx},$$

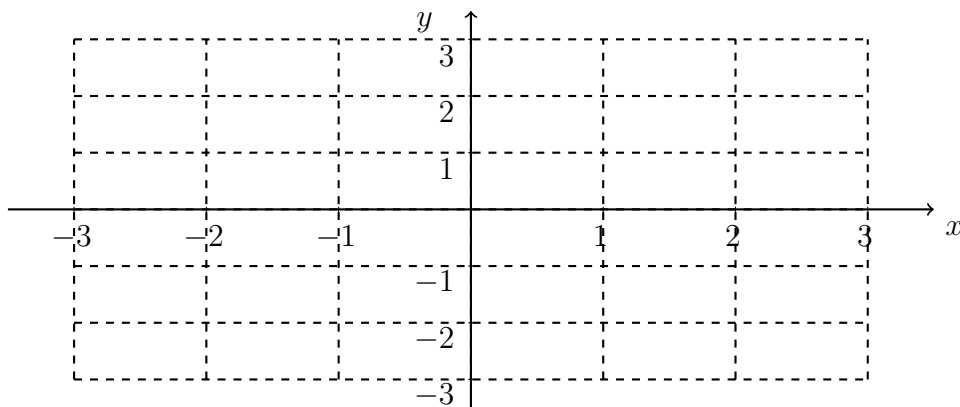
where A and r are constants.

- (a) If r is negative and A is positive what value will $p(x)$ get close to for very large values of x ? Provide a brief justification for your conclusion. If it does not get close to any value explain why.
- (b) If r is positive and A is positive what value will $p(x)$ get close to for very large values of x ? Provide a brief justification for your conclusion. If it does not get close to any value explain why.
- (c) In the case where $p(0) = 3$ and $p(10) = 4.5$ determine the values of A and r .

5. The function $k(x)$ is defined to be

$$k(x) = \begin{cases} -2(x+1) + 1 & -2 \leq x \leq -1, \\ -(3^{-x+1}) & 0 \leq x \leq 1. \end{cases}$$

(a) Use the axes below to make a sketch of the function, $k(x)$.



(b) Just by using your graph provide a brief argument as to whether or not the function is one-to-one. (Do not simply state that it passes a test but explain why the function meets the necessary conditions to be one-to-one.)

(c) Determine the inverse of $k(x)$. (If the function is not one-to-one determine a restriction on the domain and then determine the inverse on the restricted domain.)

6. The body mass of a male deer is related to its normalized antler length¹. The relationship can be approximated by

$$\text{antler length} = C \cdot (\text{body mass})^r,$$

where C and r are constants. It is estimated that a body mass of 20 kg results in a standardized antler length of 128mm, and a body mass of 30kg results in a standardized antler length of 164mm.

- (a) Determine estimates of C and r .
- (b) A deer with a mass of 25kg is captured. Determine an estimate of what its standardized antler length should be.
- (c) Determine the function that gives the estimate for the body mass of a deer given its standardized antler length.

¹Lemaître J-F, Cheynel L, Douhard F, et al. The influence of early-life allocation to antlers on male performance during adulthood: Evidence from contrasted populations of a large herbivore. *J Anim Ecol.* 2018; 87: 921–932. <https://doi.org/10.1111/1365-2656.12833>

7. The amount of a medication in a patient's blood stream decays exponentially as it is processed by the patient's liver. A patient is initially given a dose of 30mg of a medication, and after twelve hours 15mg is in the patient's blood stream. If the patient should receive a second dose when there is 10mg of the medication present, when should the next dose be administered?

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____