Division of Academic Enhancement
UNIVERSITY OF GEORGIA

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MATH 1101 Chapter 2 Review

Topics Covered

Section 2.1 Functions in the Real World

Section 2.2 Describing the Behavior of Functions

Section 2.3 Representing Functions Symbolically

Section 2.4 Mathematical Models

How to get the most out of this review:

1. **Watch the video and fill in the packet for the selected section. (Video links can be found at the two web addresses at the top of this page)**
2. **After each section there are some 'Practice on your own' problems. Try and complete them immediately after watching the video.**
3. **Check your answers with the key on the last page of the packet.**
4. **Go to office hours or an on-campus tutoring center to clear up any 'muddy points'.**

Section 2.1 Functions in the Real World

Functions
A function from A to B is a rule of correspondence that assigns each element of A to exactly ONE element of B. Functions are represented in one of three ways: table, graph or an equation (formula)

Domain (the inputs)
The domain of a function represents the set of all x values allowed to go into the function.

Range (the outputs)
The range of function represents the set of all y values allowed to come out of the function.

Tables

How do you determine if a table of values represents a function?

Notation: Domain and range are given in set notation using { } and are a list of values. The values in the sets are written in sequential order with no repeats.

Example 1
Do the following tables represent a function? State why or why not. If yes, give the domain and range.

x	1	5	3	2	5	7	10	12
y	6	1	3	3	2	4	8	25

x	2	4	5	6	9	11	3	1
y	1	2	2.5	3	4.5	5.5	1.5	0.5

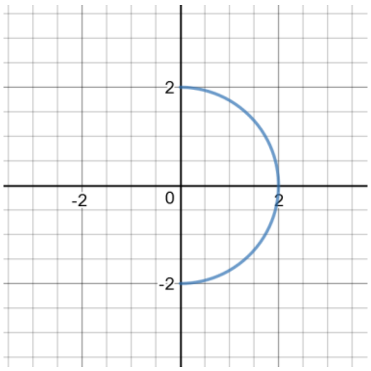
Graphs & Equations

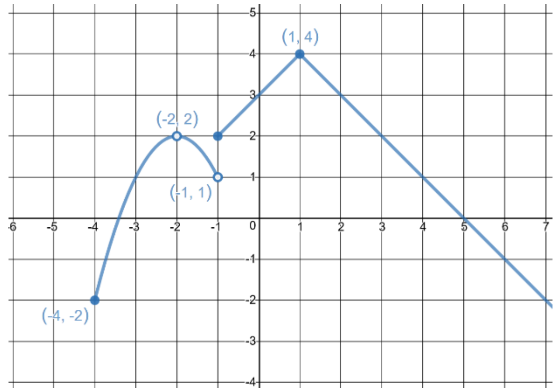
How do you determine if a table of values represent a function?

Notation: Domain and range are given in interval notation using [ ] for endpoints that *are* included in the interval and ( ) for endpoints that *are not* included in the interval. *Always* use ( ) around ∞! A combination of square brackets and parenthesis is allowed. Use ∪ (union) to connect intervals as necessary.

Example 2

Which of the following represent a relationship that is a function? State your reason for saying yes or no. If yes, give the domain and range of the function.



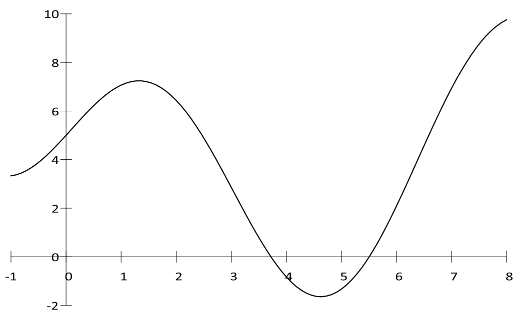


Practice on Your Own

1. Is x a function of y? Is y a function of x?

x	1	2	3	4	5	6
y	3	1	3	2	5	0

2. Is x a function of y? Is y a function of x?



3. Let N be the value of the NASDAQ stock market at the end of each day, d. Is N a function of d? Is d a function of N?
4. Find the domain and range of

x	1	2	3	4	5	6
y	3	1	3	2	5	0

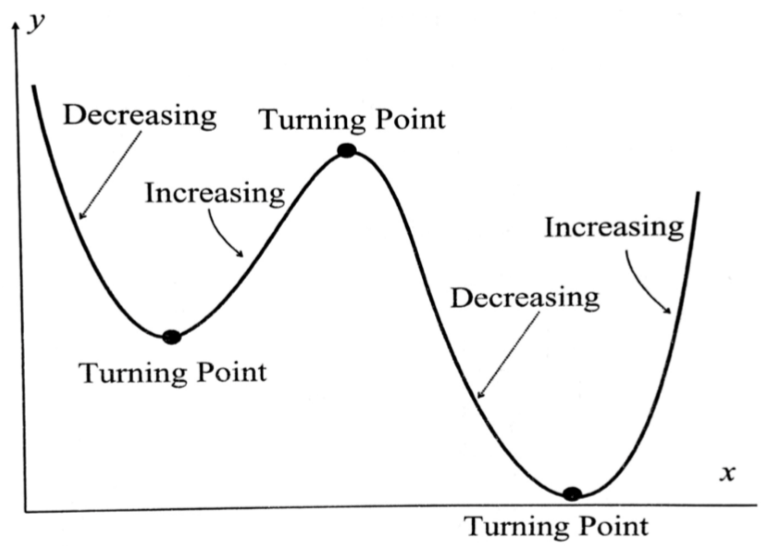
Section 2.2 Describing the Behavior of Functions

Increasing: A function is said to be increasing along an interval if the graph *rises* when read from left to right.

Decreasing: A function is said to be decreasing along an interval if the graph *falls* when read from left to right.

Constant: A function is said to be constant along an interval if the graph is *horizontal* (slope is zero) when read from left to right.

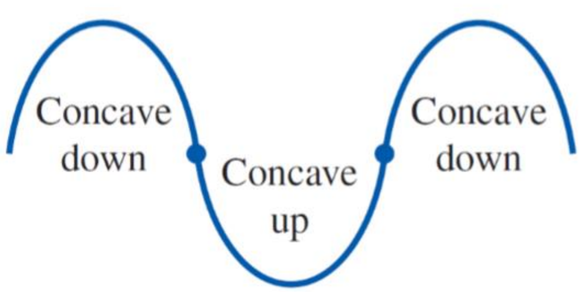
Turning Point: A function has a turning point at the location where it changes from increasing to decreasing or vice-versa. When read from left to right, the turning point is called a *local maximum* when it changes from increasing to decreasing and a *local minimum* when it changes from decreasing to increasing.

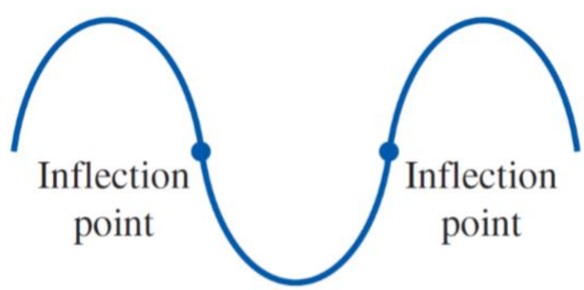


Concave Up: A function is said to be concave up if it bends upwards ('holds water') when read from left to right.

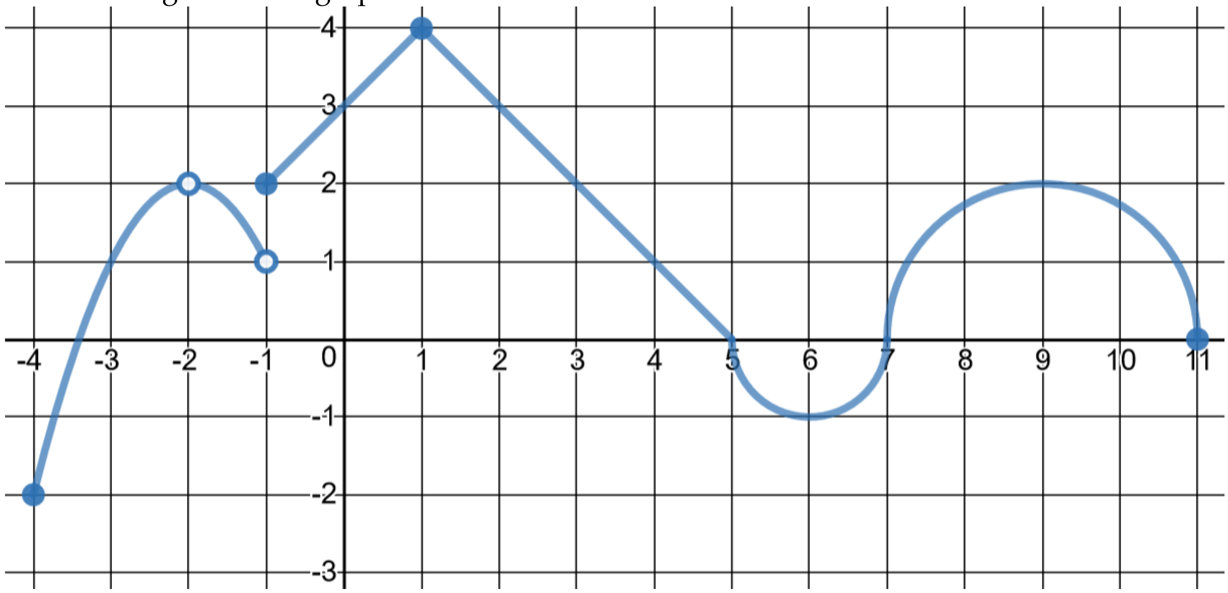
Concave Down: A function is said to be concave down if it bends downwards ('sheds water') when read from left to right.

Inflection Point: A function has an inflection point at the location where it changes from concave up to concave down or vice-versa. Inflection points occur when the function is increasing or decreasing most rapidly.





Example 3
Answer the following about the graph below.



(a) Where is the graph increasing, decreasing or constant?

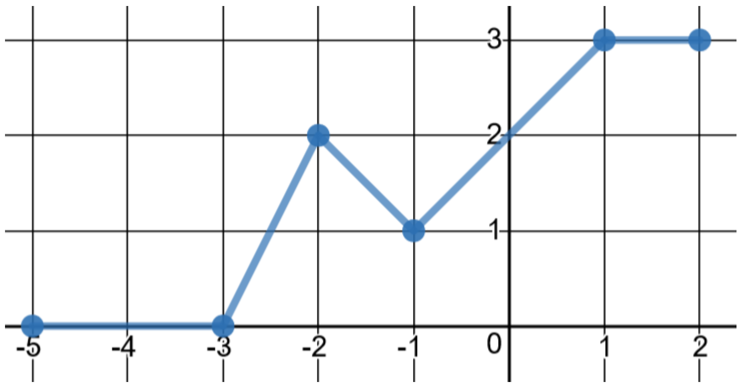
(b) Where is the graph concave up or concave down?

(c) Where are the locations of any local maxima, local minima or inflection points?

<u>Average Rate of Change (ARC)</u>
The average rate of change of a function on the interval $[a, b]$ is defined as the slope of the secant line that connects the two points on the graph. It can be calculated by

$$m = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Example 4
Calculate the ARC for the graph below along the intervals $[-5,-3]$, $[-2,-1]$ and $[1,2]$.



We can use the value of the ARC to tell us information about attributes of the graph without graphing. The following table summarizes the relationship between the ARC and behavior of the graph of the function.

Interpreting a Graph Using ARC

Sign of ARC	ΔARC (from left to right)	Description of Graph
+	Increasing	Increasing, Concave Up
+	Decreasing	Increasing, Concave Down
−	Increasing	Decreasing, Concave Up
−	Decreasing	Decreasing, Concave Down

Example 5
Use the table below to describe the behavior of the function on the following intervals.

x	y	ΔARC	x	y	ΔARC
-4	-35	35	1	0	-5
-3	0	15	2	-5	5
-2	15	1	3	0	21
-1	16	-7	4	21	43
0	9	-9	5	64	----

$[-4,-2]$

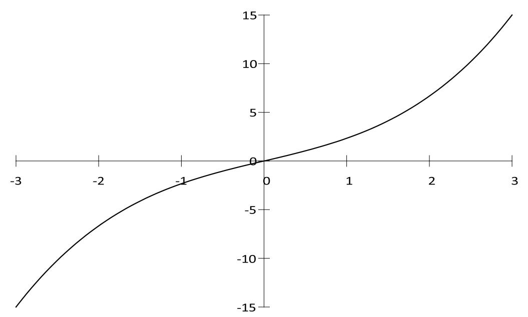
$[-1,1]$

$[-2,5]$

Are there any local maxima or minima? What interval do they exist?

Practice on Your Own

1. Use the graph to answer the following.

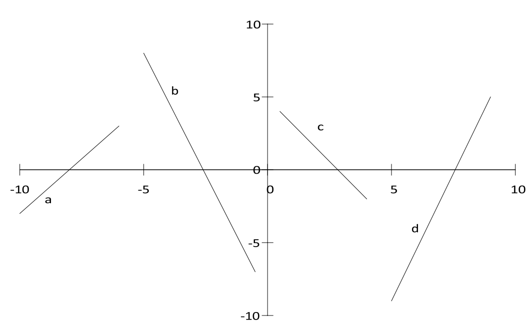


(a) Is x a function of y? Is y a function of x?

(b) In the interval $(1,3)$, the function is $[increasing\ or\ decreasing]$ at an $[increasing\ or\ decreasing]$ rate.

(c) In the interval $(-3,-1)$, the function is $[increasing\ or\ decreasing]$ at an $[increasing\ or\ decreasing]$ rate.

2. Rank the slopes of the line segments below from lowest to highest.



3. Find the average rate of change on the given intervals for the following function.
   (a) $[0,1]$

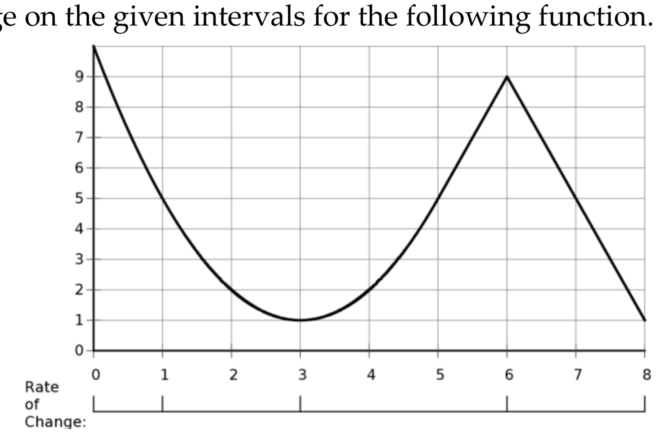
   (b) $[1,3]$

   (c) $[3,6]$

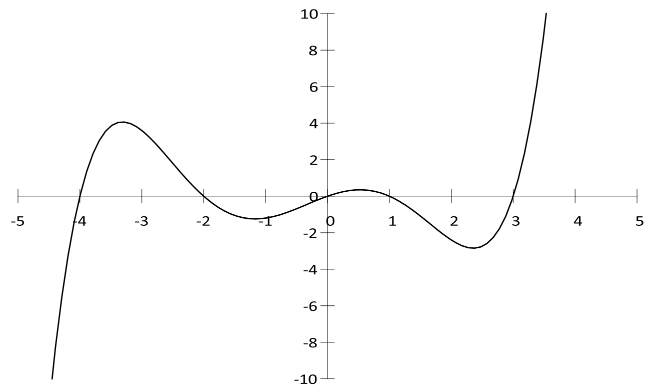
   (d) $[6,8]$

   (e) $[5,7]$

   (f) $[7,8]$



4. Use this graph to answer the following.



(a) Is this a function?

(b) What is the domain and range? (Use interval notation)

(c) How many turning points does it have? How many inflection points?

(d) How many local maxima are on the graph? Where are they located?

(e) The equation $y = -1$ has how many solutions?

(f) The equation $y = 3$ has how many solutions?

(g) Does the function have an inverse?

Section 2.3 Representing Functions Symbolically

When we write $y = f(x)$, it is read as "y as a function of x" where x represents the input variable and y represents the output variable. f is the name of the function. Note that variables can use any letter to represent them and are typically related to the quantity being measured. For example, t could be used to represent the variable time.

Finding the value of a function for a given x (input)

Option 1

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Go to 2ND→CALC→VALUE. Enter the x value and hit enter.

Option 2

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Go to GRAPH→TRACE. You can move the cursor along the graph to see various points.

Example 6

For the function $f(x) = 6x^2 + x - 1$, find $f(-1)$, $f(2.5)$ and $f(11)$.

Finding the x value of a function for a given y (output)

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Type the y value in Y_2
3. Go to 2ND→CALC→Intersect. Hit Enter three times.

Example 7

For what x value(s) does the function $f(x) = 6x^2 + x - 1 = 9$?

Calculating a single ARC value from a given equation for an interval $[a, b]$

Option 1

1. Type the equation into the equation editor (Y=) where $Y_1 = f(x)$ and exit using 2ND→QUIT.
2. Type $(Y_1(b) - Y_1(a))/(b - a)$. You can find Y_1 using VARS→Y-VARS→Function→ Y_1

For this method, pay close attention to your use of parenthesis!

Option 2

1. Go to the list editor STAT→Edit…
2. Set $L_1 = \{a, b\}$. The curly brackets { } can be found by 2ND→ ( )
3. Set $L_2 = Y_1(L_1)$. Use Y-VARS to grab Y_1 as described above
4. Set $L_3 = \Delta List(L_2)/\Delta List(L_1)$. You can find $\Delta List$ using 2ND→LIST→OPS→ $\Delta List$

Example 8

Find the ARC for the function $f(x) = 6x^2 + x - 1$ on the interval [2,4].

Section 2.4: Mathematical Models

A mathematical model is an equation that best fits a set of observed data points. The model CANNOT predict values outside of the domain of the original data set.

If the data set is not available and the model is given as an equation, pay attention to the context for the model. For example, do negative values make sense? Very small or large values?

Example 9
The equation $G(x) = 8.98x + 11.54$ models the expected grade on an exam based on the number of hours the student studied. Answer the following:

(a) What is the expected grade for a student who studied 5 hours?

(b) If the application domain is [0.10], what is the range?

(c) How many hours should a student study to expect a grade of 95?

Practice on Your Own

1. The population of Makebelievia is $p(t) = -13t^2 + 156t + 668$ where t is years after January 1, 2000. This equation works from Jan 1, 2000 to Jan 1, 2014.
   (a) What is the domain of the function?
   (b) What is the range?
   (c) How many turning points does it have? How many inflection points?
   (d) Is this function concave up, concave down or does it have a change in concavity?
   (e) Estimate the maximum population of Makebelievia.
   (f) Complete the table

t	$p(t)$	ARC
0	______	______
2	______	______
4	______	______
5	______	______
8	______	______
10	______	______
12	______	______
14	______	Blank

Answers to the Practice on Your Own problems

Section 2.1

1. x is NOT a function of y. y IS a function of x.
2. x is NOT a function of y. y IS a function of x.
3. N IS a function of d. d is NOT a function of N.
4. Domain: {1,2,3,4,5,6} Range: {0,1,2,3,5}

Section 2.2

1. (a) x IS a function of y. y IS a function of x.
   (b) Increasing, Increasing
   (c) Increasing, Decreasing
2. B,C,A,D
3. (a) $[0,1] = -5$
   (b) $[1,3] = -2$
   (c) $[3,6] = 8/3$
   (d) $[6,8] = -4$
   (e) $[5,7] = 0$
   (f) $[7,8] = -4$
4. (a) Yes
   (b) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$
   (c) 4 turning points, 3 inflection points
   (d) 2 local maxima at $(-3.3,4)$, $(0.5,0.5)$
   (e) 5
   (f) 3
   (g) No.

Section 2.3 & 2.4

1. (a) Domain: $[0,14]$
   (b) Range: $[304,1136]$
   (c) 1 turning point, no inflection points
   (d) Concave down
   (e) Max: 668

   (f)

t	$p(t)$	ARC
0	668	130
2	928	78
4	1084	26
5	1136	−26
8	1084	−78
10	928	−130
12	668	−182
14	304	Blank