By providing my signature below, I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (print): ____________________________  Name (sign): ____________________________

Student ID (81#): ____________________________  Class Time: ____________________________

Instructor's Name: ____________________________

- If you need extra space use the last page. Do not tear off the last page!
- Please show your work. An unjustified answer may receive little or no credit.
- If you make use of a theorem to justify a conclusion, then state the theorem used by name.
- Please provide neat, organized work to ensure partial credit.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Cell phones and smart watches are NOT allowed; smart devices (including smart watches and cell phones) may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.
- You are only allowed to use a TI-30XS Multiview calculator; the name must match exactly. No other calculators are permitted, and sharing of calculators is not permitted.
- You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem.

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Points Possible</th>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Total:</td>
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</tbody>
</table>
Formula Sheet

- Circles
  - area: $A = \pi r^2$
  - circumference: $C = 2\pi r$
  - Equation of the circle of radius $r$ centered at $(h, k)$: $(x - h)^2 + (y - k)^2 = r^2$

- Rectangles
  - area: $A = lw$
  - perimeter: $P = 2l + 2w$

- Cylinder
  - volume: $V = \pi r^2 h$
  - surface area: $S = 2\pi r^2 + 2\pi rh$ (includes base and lid)

- Rectangular prisms
  - volume: $V = lwh$
  - surface area: $S = 2lw + 2wh + 2lh$ (includes top and base)

- Circular cone
  - volume: $V = \frac{1}{3} \pi r^2 h$

- Sphere
  - volume: $V = \frac{4}{3} \pi r^3$
1. Determine the following limits; show work or briefly explain your thinking on each one. If you apply L’Hopital’s Rule, indicate where you have applied it and why you can apply it. If your final answer is “does not exist,” $\infty$, or $-\infty$, briefly explain your answer. (You will not receive full credit for a “does not exist” answer if the answer is $\infty$ or $-\infty$.)

(a) [4 pts] $\lim_{x \to 2} \frac{\sqrt[3]{x^2 - 6x} - 7}{x - 2}$

\[
= \frac{\sqrt[3]{4 - 12 - 7}}{2 - 2}
\]

(b) [6 pts] $\lim_{x \to 0} \frac{\sin(x) - x}{1 - \cos(x)} \rightarrow \frac{0}{0}$

\[
\lim_{x \to 0} \frac{\cos(x) - 1}{\sin(x)} \rightarrow \frac{0}{0} \\
\lim_{x \to 0} \frac{-\sin(x)}{\cos(x)} = \frac{0}{1} = 0
\]
(c) \[ \lim_{{x \to \infty}} e^{-4x} + \ln(x) = 0 + \infty = \infty \]

(d) \[ \lim_{{x \to 0^+}} \left( \frac{1}{x} + \frac{5}{x(x-5)} \right) = \infty - \infty \]

\[= \lim_{{x \to 0^+}} \frac{x - 5 + 5}{x(x-5)} \]

\[= \lim_{{x \to 0^+}} \frac{x}{x(x-5)} \]

\[= \lim_{{x \to 0^+}} \frac{1}{x-5} \]

\[= -\frac{1}{5} \]
2. [10 pts] Use the limit definition of the derivative to determine the derivative of \( f(x) = \sqrt{x+1} \).

Your answer should be \( f'(x) = \frac{1}{2\sqrt{x+1}} \). No points will be awarded for the application of differentiation rules (and L'Hopital's Rule is not allowed.) Show all steps.

\[
\begin{align*}
\frac{df}{dx} &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\
&= \lim_{h \to 0} \frac{x+h+1 - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h \to 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\
&= \lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \\
&= \frac{1}{2\sqrt{x+1}}
\end{align*}
\]
3. The graph below is the graph of the derivative of $f$ (and not the graph of $f(x)$). Use it to answer the questions that follow, keeping in mind that the graph below is the graph of the DERIVATIVE of $f$, $y = f'(x)$. The domain of the function $f(x)$ is the interval $(-4, 4)$. The grid lines are one unit apart.

(a) [2 pts] Determine whether $f(x)$ (not the derivative) is increasing, decreasing, or neither on the interval $(-2, 0)$.

$$f'(x) > 0 \Rightarrow f \text{ is increasing}$$

(b) [5 pts] $(3, 4)$ is a point on the graph of $f(x)$. Write an equation of the line tangent to $f(x)$ at the point $(3, 4)$.

slope: $-2$

$$y - 4 = -2(x - 3) \quad \text{or} \quad y = -2x + 10$$

(c) [5 pts] Determine the $x$-coordinate(s) of all relative (local) maxima of $f(x)$ on the interval $(-4, 4)$. Use $f'(x)$ to justify your answer.

$$f'$$

relative max @ $x = 2$
4. Determine \( \frac{dy}{dx} \) for each equation below. Remember to use correct notation to write your final answer. You do not need to simplify your final answer.

(a) [5 pts] \( y = \frac{2}{x^5} - 3x = 2x^{-5} - x^{\frac{2}{3}} \)
\[
\frac{dy}{dx} = -10x^{-6} - \frac{1}{3}x^{-\frac{2}{3}}
\]

(b) [6 pts] \( y = \frac{e^{-5x}}{\sin(x^2)} \)
\[
\frac{dy}{dx} = \frac{\sin(x^2)(-5e^{-5x}) - e^{-5x}\cos(x^2)(2x)}{(\sin(x^2))^2}
\]

(c) [6 pts] \( y = \ln(x) \tan(x) \)
\[
\frac{dy}{dx} = \ln(x) \sec^2(x) + \tan(x) \left( \frac{1}{x} \right)
\]

(d) [8 pts] \( y = \sqrt{\cos^2(x) + 2x} \)
\[
\frac{dy}{dx} = \frac{1}{2} \left( \cos^2(x) + 2x \right)^{-\frac{1}{2}} \left( 2 \cos(x) \sin(x) + 2 \right)
\]
Determine $\frac{dy}{dx}$ for each equation below. Remember to use correct notation to write your final answer. You do not need to simplify your final answer.

(e) [10 pts] $x^3 + y^3 = 2x + x \ln(y)$

$$3x^2 + 3y^2 \frac{dy}{dx} = 2 + x \left( \frac{1}{y} \right) \frac{dy}{dx} + \ln(y)$$

$$3y^2 \frac{dy}{dx} - \frac{x}{y} \frac{dy}{dx} = 2 + \ln(y) - 3x^2$$

$$\frac{dy}{dx} \left( 3y^2 - \frac{x}{y} \right) = 2 + \ln(y) - 3x^2$$

$$\frac{dy}{dx} = \frac{2 + \ln(y) - 3x^2}{3y^2 - \frac{x}{y}}$$

(f) [5 pts] $y = \int_2^x \left( \sqrt{e^t + \frac{1}{1 + 4t^2}} \right) dt$

$$\frac{dy}{dx} = \sqrt{e^x} + \frac{1}{1 + 4x^2}$$
5. Evaluate the following. You do not need to simplify your answers.

(a) [8 pts] \[
\int \left( 5 + \frac{1}{3x^{4/5}} - \frac{9}{x + x^2} \right) \, dx = \int \left( 5 + \frac{1}{3} x^{-4/5} - \frac{9}{x} + x^2 \right) \, dx
\]

\[
= 5x + \frac{1}{3} x^{4/5} + q \ln |x| - \frac{q}{3} + \frac{q}{3} + C
\]

(b) [8 pts] \[
\int \left( \sin(x) - \frac{1}{\sqrt{1-x^2}} \right) \, dx
\]

\[
= -\cos(x) - \arcsin(x) + C
\]

or

\[
= -\cos(x) + \arccos(x) + C
\]
Evaluate the following. You do not need to simplify your answers.

(c) \[ [6 \text{ pts}] \int \frac{3 \cos(\ln(x))}{x} \, dx \]

\[ u = \ln(x) \]
\[ 1 + du = \frac{1}{x} \, dx \]

\[ = \int 3 \cos(u) \, du \]
\[ = 3 \sin(u) + C \]
\[ = 3 \sin(\ln(x)) + C \]

(d) \[ [8 \text{ pts}] \int_{-1}^{0} e^{-3x+1} \, dx \]

\[ u = -3x + 1 \]
\[ du = -3 \, dx \]
\[ -\frac{du}{3} = dx \]
\[ u(-1) = -3(-1) + 1 = 4 \]
\[ u(0) = 1 \]

\[ \int e^u \, du = -\frac{1}{3} \left[ e^u \right]_4^0 \]
\[ = -\frac{1}{3} e^4 + \frac{1}{3} e^0 \]

\[ = -\frac{1}{3} e + \frac{1}{3} e^4 \]
6. Suppose $P(t)$ is the price of a stock, in dollars, $t$ days after the company goes public. Although we've lost the data for the actual stock prices, we did manage to find data for the rate of stock price change $P'(t)$ for the first 10 days since the stock opened!

That data is graphed below. Grid markings represent 1 unit.

Graph of $P'(t)$: the DERIVATIVE of $P(t)$

(a) [5 pts] On the graph above, draw 5 rectangles which represent a Riemann sum for the area between $P'(t)$ and the $t$-axis on $[0, 10]$, using right-endpoints.

\[ \Delta x = \frac{10 - 0}{5} = 2 \]

(b) [10 pts] Compute the Riemann sum that you drew in part (a). You may leave your answer unsimplified.

\[ A \approx 2(3) + 2(5) + 2(1) - 2(4) - 2(1) = 8 \]

(c) [5 pts] Using the Net Change Theorem, interpret the meaning of the integral $\int_0^{10} P'(t) \, dt$ in the context of the problem.

\[ \int_0^{10} P'(t) \, dt = P(10) - P(0) \quad \text{is the change in stock price between day 0 and day 10.} \]
7. (a) [15 pts] Sketch the graph of a function \( y = f(x) \) satisfying all of the following criteria:
- Its domain is \((-\infty, -2) \cup (-2, 1) \cup (1, \infty)\).
- \( \lim_{x \to -2^-} f(x) = -1 \) and \( \lim_{x \to -2^+} f(x) = 3 \)
- The line \( x = 1 \) is a vertical asymptote.
- \( \lim_{x \to \infty} f(x) = \infty \)
- \( \lim_{x \to -\infty} f(x) = -3 \)
- The sign chart of the first and second derivatives is below:

<table>
<thead>
<tr>
<th>interval</th>
<th>((-\infty, -2))</th>
<th>((-2, 1))</th>
<th>((1, 3))</th>
<th>((3, 5))</th>
<th>((5, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>sign (f')</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>sign (f'')</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

(b) [3 pts] On your sketch, label all asymptotes and points \((x, y)\) where relative extrema exist.
8. [12 pts] Each side of a square is increasing at a rate of \(6\) cm/sec. At what rate is the area of the square increasing when the area of the square is \(16\) cm\(^2\)? Your final answer should be a complete sentence with appropriate units.

\[
\begin{align*}
\text{picture} & \quad A = S^2 \\
\frac{dA}{dt} & = 2S \frac{ds}{dt} \\
\frac{dA}{dt} & = 2(4)(6) = 48 \text{ cm}^2/\text{sec} \\
\text{The area of the square is increasing at } & \quad \text{48 cm}^2/\text{sec when the area is } 16 \text{ cm}^2.
\end{align*}
\]
9. A cylindrical-shaped tube with radius \( r \) and height \( h \) is made of a long rolled-up sheet plus two circular lids, held closed with tape. We need to put tape around the rims of both lids, as well as one long strip of tape down the side of the tube. (See the labels on the figure on the below.)

![Diagram of a cylindrical tube with labels](image)

(a) [10 pts] Given that we have \( 20\pi \) inches of tape to seal this tube shut, write a function for the VOLUME of the tube as a function of the radius \( r \).

\[
V = \pi r^2 h \\
V(r) = \pi r^2 (20\pi - 4\pi r)
\]

\[
20\pi = 2(2\pi r) + h \\
20\pi = 4\pi r + h \\
20\pi - 4\pi r = h
\]

(b) [3 pts] Determine an appropriate domain for your function from (a). Briefly explain your reasoning.

\[
\begin{align*}
\pi r > 0, & \quad h > 0 \\
20\pi - 4\pi r > 0 \\
20\pi > 4\pi r \\
\frac{20\pi}{4\pi} > r \\
5 > r
\end{align*}
\]

Domain: \([0, 5]\) or \((0, 5)\)
(c) [15 pts] NOTE: Your work from parts (a) and (b) will not be used in this question.

The amount of tape needed to create a 700π in³ tube with radius \( r \) is given by

\[ T(r) = 4\pi r + \frac{700}{r^2}. \]

The domain of \( T(r) \) is \((0, \infty)\). Determine the radius \( r \) of the cylinder that will minimize the amount of tape needed. Justify, using methods of calculus, why your dimensions yield the absolute minimum.

\[
T'(r) = 4\pi - \frac{1400}{r^3}
\]

\[
= 4\pi r^3 - 1400 \quad \frac{r^3}{r^3}
\]

\( T'(r) \) does not exist at \( r = 0 \), not in the domain.

\[
T'(r) = 0:
\]

\[ 4\pi r^3 - 1400 = 0 \]

\[ 4\pi r^3 = 1400 \]

\[ r^3 = \frac{1400}{4\pi} \]

\[ r = \sqrt[3]{\frac{1400}{4\pi}} \approx 4.8 \]

Optimization Method:
- First Derivative Test or 2nd Derivative Test

\[ T' \quad \frac{3}{4\pi} \quad 1 \quad + \]

\[ T \quad \frac{3}{4\pi} \quad \sqrt{\frac{1400}{4\pi}} \quad 2 \]

The radius must be \( \sqrt[3]{\frac{1400}{4\pi}} \) in to minimize the amount of tape needed.
This page is extra space for work. **Do not detach this page.** If you want us to consider the work on this page, you should print your name, instructor and class meeting time below. For the problems where you want us to look at this work, please write "see last page" next to your work on the problem page so that we know to look here.

Name (print): ___________________________ Instructor (print): ___________________________

Class Meeting Time: ___________________________