fop

Preliminary Examination in Topology
September, 1994

1. Suppose that $X \subseteq Y$ and X is a deformation retract of Y. Show that if X is path connected then Y is path connected.

2. Prove that a set X is compact if and only if every family of closed sets in X which has the finite intersection property has a nonempty intersection. (We say that a family F has the finite intersection property if any finite collection of members of F have a nonempty intersection.)

3. Prove that any compact Hausdorff space is normal.

4. Find all surfaces (orientable or not) which can be covered by a surface (compact 2-manifold without boundary) of genus 2.

5. Give an example of a space X such that $H_n(X) \ncong \pi_n(X)$ for some n. Justify your calculations of the homology and homotopy groups.

6. Give the cell decompsotion for $\mathbb{C}P^n$. Show the attaching maps in detail. Use this to compute the homology.

7. Suppose that X is a compact orientable manifold without boundary of odd dimension. Show that the Euler characteristic of X is zero.

8. Let $\mathbb{D} = \{z \in \mathbb{C} \mid |z| \leq 1\}$ be the closed 2-disc. Let $X = \mathbb{D}/\sim$ be the quotient space where $\sim$ is the relation on $\partial\mathbb{D}$ which identifies

$$e^{i\theta} \sim e^{i(\theta+2\pi/3)} \sim e^{i(\theta+4\pi/3)}$$

for all θ.

(a) Find $H_*(X)$
(b) Find the universal covering space for X and compute $\pi_1(X)$.

9. Sketch a proof that if $f : S^n \to S^n$ is a reflection in one of the coordinates of the n-sphere then the induced map

$$H_n(f) : H_n(S^n) \to H_n(S^n)$$

is multiplication by -1.