1785

UGA High School Varsity Math Tournament
October 25, 2025

Written test

Time: 90 minutes
Length: 25 problems

Max Score 250 points
10 points for a correct answer,
0 points for an incorrect answer,
and 2 points for an answer left blank.

No calculators, slide rules, or any other such instruments are allowed.
Scratchwork may be done on the test and on the three blank pages at the end.
Credit will be given only for answers marked on the scantron sheet.
If you finish the exam before time is called, turn in your scantron sheet to the person in the front and then exit quietly.
If you need another pencil, more scratch paper, or require other assistance during the exam, raise your hand.
You will receive 10 points for a correct answer, 0 points for an incorrect answer, and 2 points for an answer left blank.

Problem 1. On this exam, you will receive 10 points for a correct answer, 0 points for an incorrect answer, and 2 points for an answer left blank. What is the total number of possible overall scores?

(A) 100 (B) 110 (C) 115 (D) 120 (E) 125

Problem 2. The underbrace notation means that a sequence of digits is repeated. For example, $5\underbrace{28}_{5}67 = 5,282,828,282,867$ and $\underbrace{1,272}_{2} = 12,721,272$. If we define

$$\Omega = \underbrace{\underbrace{123}_{123}\underbrace{456}_{456}\underbrace{789}_{789}}_{123456789},$$

what is the remainder of the division of Ω by 11?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

Problem 3. Any line in Euclidean space can be described as the set of points of the form

$$(a+\alpha t, b+\beta t, c+\gamma t)$$

where $a, b, c, \alpha, \beta, \gamma$ are constants, not all of α, β and γ are 0 and t varies over $\mathbb{R}$. John traces a line on his computer and realizes that all points $P(x, y, z)$ on it satisfy two properties: (i) the coordinates seem to satisfy some Fibonacci like rule, i.e. $x + y = z$ and (ii) the middle coordinate is always the average of the extreme coordinates, i.e. $y = \frac{x+z}{2}$. Using the notation above, what is $\frac{\beta+\gamma}{\alpha}$?

(A) 0 (B) 2 (C) $\frac{5}{2}$ (D) $\frac{1+\sqrt{5}}{2}$ (E) 5

Problem 4. For the New Year, the teacher copies all numbers from 1 to 2025 on a large blackboard. He tells the students that they are allowed to erase any two numbers provided that they write on the board the result of the expression

$$a \times b + a + b$$

where a and b are the two numbers erased. Notice that by doing so, the total amount of numbers on the boards decreases by one.

Alice begins the process by erasing 10 and 20 which she replaces by 230. The other students take turns repeating this procedure until only a single number remains. What is that number?

(A) 2025×2026. (B) 2026^{1013}. (C) $2026! - 1$.

(D) $\frac{4050!}{2025!}$ (E) The number is variable.

Problem 5. A regular octagon is built so that four of its corners coincide with the four corners of a square of area 1. What is the area of the octagon?

Ⓐ $\sqrt{2}$ Ⓑ $\frac{3\sqrt{2}}{4}$ Ⓒ $4\sqrt{2}-4$ Ⓓ $2\sqrt{3}-\sqrt{2}$ Ⓔ Some other constant.

Problem 6. What is the radius of the largest circle included in the region of the plane defined by

$$|x+|y|| \leq 1$$

Ⓐ $\frac{1}{\sqrt{2}}+1$ Ⓑ $\sqrt{2}-1$ Ⓒ $\frac{\sqrt{2}}{3}$ Ⓓ $1-\frac{1}{\sqrt{2}}$ Ⓔ $2\sqrt{2}-2$

Problem 7. Scientists discovered a small planet with radius 30 km such that its surface is entirely covered with water. A spherical meteorite hits this planet and fully sinks in the water. The height of the water increased by 1 cm everywhere. What is the estimated radius of the falling orb?

Ⓐ $\sqrt[3]{9}\times 100\ m$ Ⓑ $100\ m$ Ⓒ $3\ km$ Ⓓ $300\ m$ Ⓔ $1\ km$

Problem 8. A robot begins a journey from the corner of a rectangular room RUME with a beginning trajectory of 45°. The robot collides with one of the two longer walls of the room at point P, turns 90°to the right, and continues ricocheting from the walls at A and T, turning 90°each time. The robot then travels until it intersects its own path at the point H (as shown in the diagram, though not to scale). At the end of its journey, the robot has enclosed the rectangle PATH, which is similar to the rectangle RUME. What is the ratio of the room's longer wall to its shorter wall?

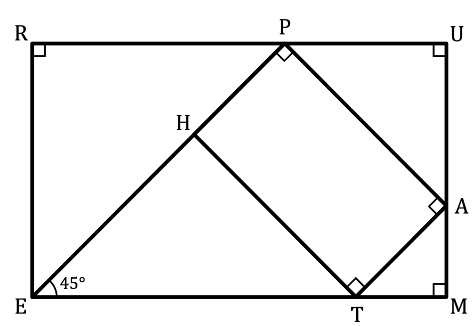

Figure 1: The journey of a robot.

Ⓐ $\sqrt{2}$ Ⓑ $\varphi=\frac{1+\sqrt{5}}{2}$ Ⓒ e Ⓓ π Ⓔ $2-\sqrt{2}$

Problem 9. We call a date Schur if the sum of the month number and the day of the month equals the last two digits of the year. For example, October 15, 2025 is Schur because 10+15=25. Let N be the number of Schur dates that occur between January 1, 2000 and December 31, 2999? What is the sum of the digits of N.

(A) 7 (B) 21 (C) 13 (D) 14 (E) 16

Problem 10. Consider the infinite table below.

$$\begin{array}{cccccc}
4 & 7 & 10 & 13 & 16 & \dots \\
7 & 12 & 17 & 22 & 27 & \dots \\
10 & 17 & 24 & 31 & 38 & \dots \\
13 & 22 & 31 & 40 & 49 & \dots \\
16 & 27 & 38 & 49 & 60 & \dots \\
19 & 32 & 45 & 58 & 71 & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}$$

It is built as follows: the top left entry is 4, the first column is the transpose of the first row, each row is an arithmetic progression, the first row has an increment of 3 and each row has increment of two more than the previous row.

If you start from number 4, go down one step to 7, go to the right one step to 12, go down to 17, etc. repeat those steps until you have moved 100 times down and 100 times to the right. What entry will you read in the corresponding box?

(A) $5,151$ (B) $10,100$ (C) $20,000$ (D) $20,200$ (E) $20,604$

Problem 11. What is the sum of the first 3 digits after the period in the decimal expansion of $\sqrt{2026}$?

(A) 2 (B) 5 (C) 7 (D) 11 (E) 19

Problem 12. Harry writes on a piece of paper the result of

$$\frac{1000}{2.014 - 1.007^2}.$$

Unfortunately, he leaves the paper in his pocket and all he can read after it goes through the washing machine is

1000.???00240111...

The three first digits after the period have become non legible; what is the sum these three digits?

Ⓐ 0 Ⓑ 7 Ⓒ 8 Ⓓ 13 Ⓔ 15

Problem 13. A *Lissajous curve* is a plane curve defined parametrically by

$$x(t) = \sin(at + \delta), \qquad y(t) = \sin(bt),$$

In other words, to each real number t we plot the point of coordinates $(x(t), y(t))$ in the plane.

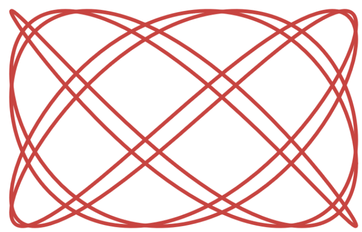

Figure 2: The Lissajous curve $(\sin(5t), \sin(7t + 1))$.

At how many points does the Lissajous curve $(\sin 4t, \sin 3t)$ intersect itself?

Ⓐ 9 Ⓑ 12 Ⓒ 16 Ⓓ 17 Ⓔ 25

Problem 14. Is the number $2024^5 + 2025$ prime?

Ⓐ Yes
Ⓑ No, it is a multiple of 3
Ⓒ No, it is a multiple of 5
Ⓓ No, it is a multiple of 11
Ⓔ No, it is a multiple of another prime

Problem 15. A robot moves in a rectangular room and everytime he hits a wall, he bounces off, so that the angle of incidence equals the angle of reflection, and continues his expedition. This time, the robot begins its journey on the long side 3 feet away from the corner with an initial angle of 45^o with the wall, and away from that corner – as is depicted below. How long will it take before he gets back to his initial position if the dimensions of the room are 35 feet by 14 feet?

Ⓐ 140 Ⓑ $105\sqrt{2}$ Ⓒ $140\sqrt{2}$ Ⓓ 210 Ⓔ 377

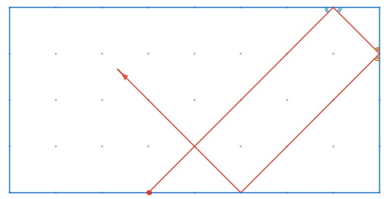

Figure 3: Another robot.

Problem 16. Let $\lfloor x \rceil$ be the integer nearest to x, with the convention that if we round up if the fractional part of x is $\frac{1}{2}$. E.g. $\lfloor 4.87 \rceil = 5$, $\lfloor 4.33 \rceil = 4$ and $\lfloor 4.5 \rceil = 5$. Evaluate the following sum:

$$\frac{1}{\lfloor \sqrt{1} \rceil} + \frac{1}{\lfloor \sqrt{2} \rceil} + \frac{1}{\lfloor \sqrt{3} \rceil} + \cdots + \frac{1}{\lfloor \sqrt{2024} \rceil} + \frac{1}{\lfloor \sqrt{2025} \rceil}$$

(A) 84 (B) 87 (C) $\frac{191}{2}$ (D) 89 (E) 90

Problem 17. Evaluate

$$\sum_{n=1}^{2025} \# \left\{ (x, y) \in \mathbb{N}^2 \mid \frac{1}{x} + \frac{1}{y} + \frac{2^n - 1}{xy} = 1 \right\}.$$

where # denotes the number of elements in a set.

(A) 205,3350 (B) 205,3351 (C) 205,420 (D) 212,314 (E) 212,420

Problem 18. Let $M(x) = \lfloor \sqrt{x} \rfloor$ where $\lfloor x \rfloor$ denotes the *floor* function, i.e. the largest integer $n \leq x$. E.g. $\lfloor \pi \rfloor = 3$, $\lfloor 7 \rfloor = 7$ and $\lfloor -4.4 \rfloor = -5$.
How many integer solutions does the equation

$$M(M(M(x))) = 4$$

have?

(A) $325,089$ (B) $325,090$ (C) $325,091$ (D) $325,092$ (E) Some other number.

Problem 19. Imagine a non degenerate triangle whose interior angles are α, β and γ and such that the tangent of these angles form a geometric sequence,

$$\tan \alpha = r \tan \beta = r^2 \tan \gamma$$

where $r \geq 1$. What is the largest possible value of $\tan^2(\gamma)$?

Ⓐ $\frac{1}{3}$ Ⓑ 2 Ⓒ $\frac{3}{2}$ Ⓓ 3 Ⓔ $\frac{7}{2}$

Problem 20. A point P is located at a distance of 4 from the center of a circle C of radius 2. A line m is drawn through P and through two points Q and R on C, distant of 2 from one another. Consider the circle C_Q which is tangent to C at Q and passes through P; and the circle C_R which is tangent to C at R and passes through P. What is the difference, in absolute value, between the diameters of the circles C_Q and C_R?

Ⓐ 2 Ⓑ 3 Ⓒ $2\sqrt{2}$ Ⓓ 4 Ⓔ $3\sqrt{2}$

Problem 21. Recall the table of Problem 10.

$$\begin{array}{cccccc}
4 & 7 & 10 & 13 & 16 & \dots \\
7 & 12 & 17 & 22 & 27 & \dots \\
10 & 17 & 24 & 31 & 38 & \dots \\
13 & 22 & 31 & 40 & 49 & \dots \\
16 & 27 & 38 & 49 & 60 & \dots \\
19 & 32 & 45 & 58 & 71 & \dots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{array}$$

What is the smallest number stricly larger than 2025 not appearing in this table?

Ⓐ 2028 Ⓑ 2031 Ⓒ 2067 Ⓓ 2068 Ⓔ 2069

Problem 22. What is the coefficient of x^{60} in the polynomial

$$(x-1)(x^2-1)(x^3-1)\cdots(x^{15}-1)?$$

Ⓐ 0 Ⓑ 120 Ⓒ 722 Ⓓ 60 Ⓔ 15!

Problem 23. Consider the set S of integral points in the square $\mathcal{O}(0,0)$, $A(0,25)$, $B(25,25)$ and $C(25,0)$, i.e.

$$S = \{(x,y) \mid x, y \in \mathbb{Z}; 0 \leq x, y \leq 25\}.$$

We say that the points P and Q in S are *friends* if the amplitude of the angle $\widehat{P\mathcal{O}Q}$ is 45^o. By convention $\mathcal{O}$ has no friends. Let F be the subset of points in S which have a friend. How many elements does F have?

Ⓐ 529 Ⓑ 539 Ⓒ 549 Ⓓ 559 Ⓔ 569

Problem 24. For his birthday, Alice's teacher copies all numbers from $1, 2, \cdots, 2026$ on a board in an arbitrary order. He tells the students that they are allowed to erase any two numbers provided that they then write their positive difference. After 2025 steps only one number is left on the board. What is the maximum possible value for this number?

(A) 1012 (B) 1013 (C) 2001 (D) 2026 (E) 2025

Problem 25. The numbers n and $n+2$ are positive integers whose product, when divided by 2026, is a nonzero perfect square. If n is as small as possible, what is the sum of the decimal digits of n?

(A) 2025 (B) 41 (C) 27 (D) 10 (E) 9

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Answer Sheet

Name

ZipGrade.com

2020 MC bubble (0634)

1	A	B	C	D	E	18	A	B	C	D	E
2	A	B	C	D	E	19	A	B	C	D	E
3	A	B	C	D	E	20	A	B	C	D	E
4	A	B	C	D	E	21	A	B	C	D	E
5	A	B	C	D	E	22	A	B	C	D	E
6	A	B	C	D	E	23	A	B	C	D	E
7	A	B	C	D	E	24	A	B	C	D	E
8	A	B	C	D	E	25	A	B	C	D	E

9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E

Student ID

0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9