Problems are weighted equally. Justify your answers. $\mathbb{R}$ stands for the real numbers.

1. Suppose f and g are injective maps of a set A into itself. Show that the composite function $f \circ g$ is also injective.

2. Use the ϵ, δ definition to prove that $\lim_{x \to 2} x + \frac{1}{x} = \frac{5}{2}$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = x\,\mathrm{e}^{2x}$. Find a general formula for the n'th derivative $f^{(n)}(x)$ and prove it by induction.

4. Find a basis for the subspace of $\mathbb{R}^4$ spanned by the vectors $(1,1,1,1), (3,4,6,7),$ and $(5,6,8,9)$.

5. Suppose A is a three by three matrix with real entries having eigenvalues $-1, 0,$ and 1. Prove that $A^3 = A$.

6. Find the Maclaurin series of the function $f(x) = \arctan x$ and use an appropriate Maclaurin polynomial to approximate $\arctan .01$ with an error of less than 10^{-5}.

7. Suppose (f_n) is a sequence of bounded functions taking $\mathbb{R}$ into $\mathbb{R}$ which converge uniformly to a function g. Prove that there is a number $M > 0$ satisfying $|f_n(x)| \leq M$ for all natural numbers n and all real numbers x.

8. Suppose $f : \mathbb{R}^2 \to \mathbb{R}$ has continuous partial derivatives at $(0,0)$. Prove that f has directional derivatives in every direction at the origin.

9. Describe the construction of a field having eight elements.