SYLLABUS FOR **MATH 2200**

Text: Larson and Edwards,

Calculus of a Single Variable, Early Transcendental Functions,

Fifth Edition (2011)

(Custom Edition for UGA)

Effective Summer, 2011

Introductory Words

The following description of the objectives of the course as described in CAPA:
This is an introductory calculus course. Students should understand the concept of limit and the meaning and import of continuity. The student should understand the concept of the derivative and be able to calculate the derivatives of algebraic, exponential, logarithmic, and trigonometric functions. Students are expected to set up and solve maximum and minimum problems using the methods of calculus. Students should be able to use calculus to make approximations and sketch graphs. Students should also be able to calculate antiderivatives and solve some elementary differential equations. The student should in addition understand the meaning and application of the derivative in the context of economics.

The Curriculum Committee believes that the following five topics are central to the course and should be covered (in depth) by everyone teaching the course.

(1) The definition of the derivative: calculating it from the definition, its meanings (slope of the tangent line, velocity, rate of change)
(2) Differentiation rules (including facility with the chain rule)
(3) Reading, setting up, and solving word problems (extremum problems and related rates)
(4) Conceptual skills with graphing—understanding the first and second derivative
(5) Some experience with the "theoretical infrastructure" of calculus—some discussion of any or all of the following: continuity and the intermediate value theorem, maximum value theorem, mean value theorem.
(6) Basic antidifferentiation with applications to separable differential equations and the mortgage problem

TEXT CONTENT (in recommended order): Sections 1.1–1.6: optional or review in context; sections 2.1–2.4, 3.1–3.5, 3.7, 4.1–4.4, 2.5 with 4.5, 4.6, 4.7, 4.8, Appendix F, 5.1, 6.2–6.3

ANCILLARY MATERIALS: The text comes with a web address for graphs from exercises to use for handouts. www.mathgraphs.com The Power CD is available with PowerPoints of lecture materials from the text (Julie McEver has this with the text). WebAssign provides an opportunity to avoid purchasing the text: links within individual exercises takes you to the relevant section of the text. In addition, the Resources tab in student view will allow students to watch a publisher-provided video for each section in our course syllabus.

Guidelines for grades

- **A:** The student has computational mastery of the course, can set up and solve non-routine word problems, and has an understanding of the theoretical aspects of the course (e.g., how to apply the Intermediate Value Theorem or Mean Value Theorem).
- **B:** The student has computational expertise, but may make occasional errors, can set up and solve standard word problems, and, e.g., can give an example of a non-differentiable continuous function.
- **C:** The student demonstrates basic computational skills, has some conceptual understanding of the meaning of the derivative, and can do most of a routine word problem.
- **D:** The student can do routine calculations, including a moderate chain rule application, can find the equation of the tangent line, but struggles to set up a routine word problem.

Grade distribution The expected grade distribution in MATH 2200 is approximately the following:

grade	percentage
A	27
B	28
C	20
D	7
F/WF	5
WP	13

Outline of course

This syllabus is based on a MWF schedule. TR classes should cover 3 days' worth in 2 days. If you are a grad student, you may stretch out lectures into recitation and put the HW/Quiz activity on other days as well. For teaching faculty, the syllabus is rather tight, so you probably want to hold off HW discussions for your TA to do at recitation.

Recitation instructors should be prepared to present the problems. Instructors should let their recitation instructors know ahead of time of any special conventions, approaches, or other deviations from the standard syllabus.

We recommend in this syllabus that you present certain examples from the text. However, it is more effective if you use these as guides, making small changes and not just presenting the text *verbatim*.

Keep algebra simplified—don't multiply out unless absolutely necessary. Set a good example for your students. Resist the temptation to be too fancy or too rigorous. If you have particularly curious students, of course, you can encourage them with more sophisticated tidbits out of class. (For example, a "simple" function that is curiously discontinuous is given by $f(x) = [x] + [-x]$.)

(This is 42.5 class days (50 min) with no review days inserted)

I. Prelude to calculus (2 1/3–2 2/3 weeks)

1.1–1.6 Review Concepts from Precalc, Intro to WebAssign HW (1–2 days)
2.1 Intro to Differential Calculus (boxes p. 63 and p. 65 tangent line problem) (1/2 day)
2.2 Intro to Limits: Graphical and Numeric Understanding, including one-sided limits (from 2.4) (NO ϵ-δ) (1 day)
Examples 2.2: 1, 3, 4, 5 and 2.4: 2, 3
Exercises (corresponding to those created in WebAssign) 2.2: #1, 2, 5, 7, 6, 19, 20, 21, 22, 25, 27, 28, 30, 31; 2.4: #14, 124, 33, 36
2.3 Symbolic work on Limits, including Trig and Squeeze, velocity as a limit (2.5 days)
Examples 2.3: 1, 2, 3, 5, 7, 8, 9, 10, and #68 in HW
Exercises 2.3A: #5, 7, 9, 12, 18, 19, 21, 20, 29, 32, 34, 38, 40 53, 41, 46;
2.3B: # 50, 58, 60, 65, 67, 70, 71, 75, 77, 91, 94, 96, 109, 111
2.4 The Concept of Continuity and IVT (2 days)
Examples 1, 2 and 4, 3, p. 101 #107 and 108, 7, Figure 2.26, 8, p. 102 #119 or 120 (be sure to define a function $f(x)$ and verify that the Intermediate Value Theorem applies)
Exercises 2.4: #2, 5, 86, 87, 33, 39, 42, 43, 52, 18, 58, 59, 62, 69, 73, 103, 106, 118

II. The derivative: Rules of differentiation (3 2/3 weeks)

3.1 The Derivative and Tangent lines (2 days)
Examples 2, 1, 3, 5
Exercises 3.1A: #1, 16, 18, 21, 23, 3.1B: #4, 60, 43, 20, 23, 25, 30, 33, 61, 93, 40, 84, 88

——EXAM 1——

3.1 When the derivative fails to exist (1/2 day)

Examples 3.1 material just before Ex. 6, and 6 and 7, p. 124 #39–42 and 45–50 (choose some)

3.2-3.3 Basic Differentiation Rules, higher order derivatives, and Rates of Change (3.5 days)

Examples 3.2: 1, 2, 4, 6, 8, 9, 10, 11, p. 137 #76, 105, 107;
3.3: 1, 2, 4, 5, 6, 8, 10 p. 148 #85, 87, 120

Exercises 3.2: #2, 4, 6, 9, 13, 21, 27, 30, 34, 40, 41, 45, 49, 59, 64, 68, 72, 102, 101, 104, 112, 114;
3.3: #109, 111, 112, 83, 39, 54, 7, 12, 16, 21, 25, 28, 46, 49, 62, 95, 96, 104, 67, 70, 76, 85, 89

3.4 The Chain Rule (2 days)

Examples 1, 2, 4, 5, 6, direct students to read 7–9 (not useful until solving $f'(x) = 0$), 10, 11, 12 and p. 163 #148, 190 and 154

Exercises 3.4: #1, 10, 15, 24, 28, 32, 34, 53, 59, 62, 68, 74, 76, 80, 84, 86, 94, 97, 109, 111, 126, 131, 143, 144, 161, 162, 168, 171

3.4 Derivatives of ln functions (1/2 day); note: $y = a^x$ and $y = \log_a x$ for bases other than e are in HW 3.4 #131, 143, 144 but not required to be memorized—for those two derivatives students are welcome to look up the rule.

Examples 3.4: 13, 14, 15, 16

Exercises in 3.4 HW above, 84, 86, 94, 97, and see note on 131, 143, 144

3.5 Implicit Differentiation (no second derivative, no logarithmic differentiation) (2 days)

Examples 1, 4 ,5 and HW exercises boxed below to illustrate product rule and work some applications

Exercises 2, [6], 7, [12], 16, 12, 19, 38, 39, 43, 49, [101], [102]

III. The derivative: First Applications (1 week)

3.7 Related Rates (2 days)

Examples 1, 3, 4 and p. 188 #24 and 42, and some from exercises

Exercises 3.7: #1, 8, 14, 13, 15, 21, 25, 28, 29, 31, 33, 46

4.1 Maxima and Minima of Functions on Closed Intervals (1 day)

Examples 2, 3, 4 and p. 210 #66-69

Exercises 4.1: #64, 1, 5, 11, 13, 15, 18, 25, 27, 31, 38, 72

——EXAM 2——

IV. Additional Applications of the Derivative: Mean Value Theorem and Curve Sketching (2 2/3 weeks)

4.2 Mean Value Theorem and its consequences (No Rolle) (2 days)

Examples 3, 4 and p. 217 #67, 72, 88 or 90

Exercises 4.2: #42, 43, 48, 50, 70, 79, 82

4.3 Increasing and Decreasing Functions and the First Derivative Test (1.5 days)

Examples 1, 2, 4

Exercises 4.3: #2, 9, 19, 25, 36, 45, 54, 62, 101, 105

4.4 Concavity and curve sketching (Omit Second Derivative Test) (1.5 days)

Examples 1, 2, 3, and p.236 #75, 78, 83

Exercises 4.4: #4, 19, 30, 38, 96 and §4.6 #17, 23, 29, 90 (note part (b) is worded wrong)

2.5 and 4.5 Limits involving infinity and asymptotes (2 days)

Examples §2.5: 1, 3, 4, 5; §4.5: 1, 2, 4, 7, 8; §4.6: 2

Exercises 2.5: #6, 5, 7, 14, 20, 27, 35, 37, 42, 50, 55, 61, 73; 4.5: #15, 18, 20, 30, 35, 98, 3, 74; 4.6: #6, 8, 98

4.6 Sketching wrap-up (1 day)

Examples 5, 3 and p. 256 #65, 79, 89

Exercises 4.3: #82, 83, 85; 4.6: #4 , 3 additional UGA-coded problems in WebAssign

V. Additional Applications of the Derivative: Optimization, Differential Approximations, and Business Applications (2 2/3 weeks)

4.7 Applied Optimization Problems (3 days)

Examples 1, 2, 4, 5 and spend class time on HW problems too

Exercises 4.7A: #5, 7, 9, 16, 17, 19, 20, 27; 4.7B: #60, 22, 23, 25, 29, 33, 39, 45, 49

——EXAM 3——

4.8 Differential Approximations (2 days)

Examples 1, 2, 7, 4, 5, 3 and p. 276 #26, 28

Exercises 4.8: #2, 9, 12, 13, 17, 18, 19, 25, 27, 30, 31, 34, 36, 37, 40, 41

Appendix F Economics Applications, including marginal cost, average cost, diminishing returns, and optimization problems (3 days)

Examples Define all terms on F1 examples 1, 3, 22, 5, 6

Exercises Appendix F: #30, 1, 25, 5, 6, 15, 9; 3.2: #114; 4.2: #34; 4.5: #96, 101; 4.7: #54; 4.8: #39, 1 additional UGA-coded problem in WebAssign

VI. Antiderivatives (2 weeks)

5.1 Antiderivatives and Initial Value Problems (including linear changes like $1/(2x-5)$ but NOT integration by substitution)(2 days)

Examples 1, Integration table p. 286 except a^x, 3, 4, 5, 6, 7, 8

Exercises 5.1A: #5, 6, 9, 11, 12, 14, 18, 20, 23, 24, 26, 29, 31, 34, 35, 38, 39, 40, 3 additional UGA-coded problems in WebAssign;
5.1B: #3, 55, 63, 66, 68, 69, 71, 72, 77, 79, 80, 89, 92, 101, 93

6.2–6.3 Separable Equations and Applications (skip p. 407-408 and p. 410) (3 days)

Examples 6.2: 1 and derivation prior to 2, 3, 6 and 6.3: 1, 3, 7, 9

Exercises 6.2: #18, 5, 7, 19, 2, 10, 13, 11, 22, 33, 41, 42, 50, 65, 69, 71, 73, 89, 90; 6.3 #1, 3, 7

——EXAM 4——

Outline of Exam Topics

These lists are not comprehensive, and each is too long for a single test. We recommend 75 minute tests if your recitation is in a 75 minute block. The test should cover both straightforward and more challenging problems from these lists, so as to distinguish among A, B, C, and D/F students.

Exam 1

(1) Limits
  (a) Limits from a graph, including one-sided limit
  (b) A limit that does not exist
  (c) Easy algebraic limit
  (d) Factoring trick
  (e) Conjugate trick
  (f) Problem involving $\lim_{x\to 0}(\sin x)/x$ or cosine version
  (g) Rate of change application: velocity as a limit
(2) Continuity
  (a) Finding points of continuity (or nondifferentiability) on a graph or verifying continuity or discontinuity for (piecewise-defined) functions
  (b) Using the Intermediate Value Theorem to prove existence of a solution of an equation (this should include defining a function and checking the hypotheses); or explaining other applications of IVT.
(3) Computation of a derivative from the definition for a simple rational function or a function involving a square root
(4) Application of tangent line—find horizontal tangents, find where tangent is parallel to another given line, use velocity to make conclusions about straight line motion, etc.

Exam 2

(1) Computation of derivatives using sum, product, quotient, generalized power rules and derivatives of trig functions, ln and exp
(2) Computation of derivatives using the chain rule
(3) Proof of $\sin' = \cos$ or $\cos' = -\sin$
(4) Match graphs of derivatives with graphs of functions or identifying where a derivative does not exist on a graph
(5) Implicit differentiation
(6) Using slope from implicit differentiation to locate points on a curve (horizontal or vertical tangents, or tangents parallel to a given line)
(7) Related rates problem (involving similar triangles or Pythagorean Theorem)
(8) Extrema of a given function on a closed interval
(9) Rate of change application (velocity, population change, etc)

Exam 3

(1) Match graphs of derivatives with graphs of functions
(2) Use sign analysis of $f'(x)$ to determine intervals on which $f(x)$ is increasing and decreasing; ditto for $f''(x)$ for concavity
(3) Use the graph of a derivative to predict inc/dec and concavity for the original function
(4) Curve sketch of an algebraic function, e.g., $f(x) = x^{4/3} \pm 4x^{1/3}$, or rational function. This includes behavior as $x \to \pm\infty$, intervals where f is increasing/decreasing, characterizing

critical points, finding vertical tangents, intervals where f is concave upward/downward, inflection points ... and sketching. This is to be done with no calculator. With a calculator, give some information and require construction of the sketch.

(5) Limits involving infinity ($\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to\infty} f(x)$)
(6) Finding horizontal, vertical, and slant asymptotes of a rational function
(7) Max/min problem on an open interval; require the students to model and provide justification
(8) Maximum/minimum word problem on a closed interval; require the students to model and provide justification

<u>Exam 4</u>

(1) Computing and explaining the meaning of marginal cost/revenue/profit
(2) Solving previous calculus problems in a business/econ context: e.g., using an infinite limit to compute long-term average cost
(3) Modeling and solving an optimization problem in business context: e.g., using data to model a linear pricing function and cost function and maximizing profit
(4) A second optimization problem: say, given cost, minimize average cost.
(5) Using a linear approximation from given function and rate values to estimate other values in context.
(6) Application of linear approximation (e.g., to estimate $\sqrt{98}$): the student must define a function and determine the appropriate $x = a$ at which to do the linear approximation
(7) Differential approximation word problem
(8) Simple antidifferentiation problems and a simple initial value problem (e.g., $dy/dx = e^{2x} + 4x^2$, $y(0) = 1$)
(9) Braking problem or determining height of building from time of impact of falling object.
(10) Separation of variables: either Newton's law of cooling or interest with deposit

<u>Final Exam</u>

The final exam should be cumulative. Repeating one or two word problems from previous tests is a good measure of whether the students retained or improved their understanding. In addition to limits, continuity, derivatives (regular and implicit) and tangents, and curve sketching, make sure you include one related rates, one differential approximation, and two optimization problems (one at C level, one for A/B students). Final exam should be about 1.5 times as long as your 75 minute tests.

Weekly Instruction Outline

Week 1 First class: Intro to WebAssign. Students self-enroll via a classkey that you give, and HW is waiting. You set due dates. To edit an assignment, you have to duplicate the assignment, make the original unavailable, then edit and save and schedule your modified assignment to your class. See helpful handout.

First class: Mini lecture from 2.1. What is *calculus*? In our case, you might (briefly) mention the integral calculus, but this course focuses on differential calculus. This section not assessed until you actually build tangent lines and compute derivatives via the definition, just prior to test 1.

(Optional) Review material from 1.1–1.3. This can be done via reading and homework assignments, or group work in recitation. This material is presumed. Students with severe gaps should consider our precalculus course. Townsley puts the definition of function on quiz—you would be surprised how many students don't know it. There are WebAssign review assignments for 1.1–1.6—whether they are required for your course grade is your decision.

The (assessed) course begins in earnest with 1.5–1.6, a review of the exponential and logarithmic functions. Their domains, ranges, graphs, and properties. Emphasize e as an intuitive limit which arises in the computation of continuously compounded interest, and then consider the inversion of $n \to \infty$ as $x = 1/n \to 0$ (p. 51 in 1.6 and again Theorem 2.9 on p. 85). Practice solving some logarithmic and exponential equations to make sure the students' foundation is solid. The inverse trig functions are NOT in the course syllabus. If you think you are pressed for time in unit 1, consider this review material also and minimize the lecture time (or reserve for recitation class only). They need log properties to efficiently differentiate later in unit 2.

End week 1 with an intro to limits (2 classes). Begin §2.1 with just a feeling for limit, using the e example from the previous lecture. Examine some graphs and compute some tables of values. Emphasize that limit is a different concept than function value (or else we wouldn't have a separate idea!). Sometimes these values coincide, and that is because the function in question happens to be "nice" there (continuous, of course). Functions that are familiar except for a "hole" (figures 2.5, 2.6, 2.7 in section 2.1) are a good demo of this. Finish with an example or two of one-sided limits. Yes, we do one-sided limits in the graph/table section even though in the text they happen in §2.4. Mention limits may not always exist—to be studied next week. Warning: Some students have difficulty reading function values from graphs, so take this slowly. ϵ-δ is in §2.1—not on the course syllabus. Townsley explains the notation so they understand it is the formal definition of "arbitrarily close", but no HW or assessment.

Week 2 (or last day of week 1) Begin with more examples of limit behavior that requires more work/thought. First, several types of limits failing to exist: (uniform) oscillation, one-sided limits not agreeing, infinite behavior. Compare $x \sin(1/x)$ to $\sin(1/x)$ to see that oscillation doesn't necessarily preclude a limit from existing. Determine limits on some piecewise defined functions. Look at the greatest integer function (calling it the "floor" and "round down" really helps) and some limits, and also x-[[x]] or such.

(Try to start week 2 here) To begin analytic limits, mention that our investigations lack rigor—what is "arbitrarily close"? Luckily, we have theorems that tell us how to be sure of our answer, the limit laws. Do the easy ones first, then work up in a subsequent lecture to add on additional algebra techniques for limits that don't yield immediately to direct substitution (conjugate trick, clearing compound fractions, etc). Illustrating that the tangent line slope is found as one of these denser algebraic manipulations will take awhile. Convince students that 0/0 limit forms can turn out to be any answer.

Later, introduce the Squeeze Theorem and use it to prove the 2 important trig limits. (You need to be careful about $\pm\theta$). Other trig limits require reciprocal identities and Pythagorean identities and getting to the 2 important trig limits. This is a good time to squeeze in derivative of sine/cosine from definition and test on test 1 to free up space on test 2 which is crowded. Recitation this week should focus on HW/calculations they are stuck on and make sure to give a quiz to keep students on task. This week is all about getting algebra ready for the rest of calculus.

The application which should be **emphasized** is finding velocity from position as a limit. There are a lot of related questions you can ask in position/velocity, see p. 89 (that our 1113 students have studied). We postpone 2.5 infinite limits to the unit on curve sketching (asymptotic behavior).

Week 3 This week you need to formalize continuity and its relationship to the limits we were computing easily before, and the consequences of continuity (IVT, existence of solutions to equations). Text has some word problems involving the floor function as application (p. 101 exercises 115–118). Last chance to give a quiz before test 1, if you test in week 4.

Begin the definition of the derivative. Emphasize the limit idea, tangent slope as limit of secant slope. Look at both forms of the derivative ($h \to 0$) which this text does as Δx, and also $x \to a$ which is useful for upcoming proofs). Do some derivatives computationally, then some by estimation on a graph. Study the 3 ways a derivative can fail to exist (vertical tangent, corner/cusp, discontinuity). Point out that we already have an application of derivatives: velocity is the derivative of position. Review how to find a tangent line. Link between continuity and differentiability.

Week 4 Derivative rules, and higher order derivatives. Find a LOT of tangent lines, including the graph applied problems (see p 125 number 61 for an example of some of the nice features that parabolas have). Other applications have the student read an application description and extract info, then find rates of change. Visit velocity again, include acceleration and "jerk." Students have difficulty generalizing beyond up/down freefall, so visit the particle on the x-axis, or E/W motorway—what is $v > 0$, $v < 0$? Week 4 or Week 5 is the time to give test 1, in the recitation. Week 4 is ideal, but can be a problem if your recitation is Tuesday, since there is a holiday (Labor Day or MLK) in there that takes away a class.

Week 5 You work through the derivative rules, with some proofs. This may begin at the end of week 4. For the x^n power rule, rather than use the Binomial Theorem with mysterious coefficients, consider factoring $a^n b^n$, and then in our case $a = x + h$ and $b = x$. Shifrin recommends the following visual argument for the product rule: Consider $(u+\Delta u)(v+$

$\Delta v) - uv$, with the corresponding picture of an expanded rectangle. Townsley does more computations than (derivative rule) proofs, but squeezes in proofs at the end of each class. Students should be responsible for one proof or theorem statement on each test (ex: proof that derivative of $\tan x$ is $\sec^2 x$). Stuff to make a big deal of: students can't see a good way to differentiate $\frac{5}{3t^2}$ or $\frac{t^3}{4}$. Also, they think the power rule applies to $\sqrt{\pi}$ and e^2. This text is "early transcendentals," so e^x and sin and cos appear early so that the power rule and quotient rule have some appropriate use. Notice that until you get to the Chain rule, $\sqrt{x+1}$ can't be done except by the definition of the derivative, and $1/(3x-4)$ needs the quotient rule (or the definition of derivative again). Applications need to be seen repeatedly, since students focus only on job at hand. So visit position/velocity/acceleration and "the rate of change of blob" once a day also. Any time you have for group work on practicing derivatives this week and next is necessary—there is a great divide between students with HS calc experience in your class (they spend 1 whole week on the power rule!) and those learning derivatives for the first time. The overarching problem is that a student can't tell you what kind of function they are looking at: constant, power, transcendental, product, etc.

Weeks 5 and 6 are tight in schedule—students start to really grapple with what is a function and how to differentiate it, and they will try to slow you down. Written assignments and/or group work is a way to assess how this is going.

Week 6 You may get to the Chain Rule by the end of week 5 (good!). The chain rule really complicates matters—the student doesn't know which rule to do when. A correct rule of thumb is to do the order of operations—forward to build the function and in reverse to perform differentiation (peeling an onion). It is important to give a no-calculator quiz on derivatives and tangent lines prior to test 2.

Applications so far are "find the rate of change of the population of chipmunks", "find the horizontal tangents, or for a parabola the two tangents that intersect at (3,2)", and "an object is tossed off boyd roof and hits the ground 2.5 sec later".

Now begin more advanced applications of differentiation, beginning with implicit differentiation and following up with related rates. Both of these are applications of the Chain Rule, and written in Leibniz's form of the derivative. So students need to get comfortable with Leibniz's notation and away from y' notation. You can introduce the derivative of $\ln x$ at the beginning of the week with the Chain Rule, and prove it later as application 1 of implicit differentiation. You need lots of class time to monitor student work on identifying the proper rule and implementing it. A former grad student always set up u, v, u' and v' (for the various rules in which they appear) in a separate table with his students, and they differentiate well this way.

Week 7 Finish up implicit differentiation with some applications (points on given ellipse with vertical tangents, points on given circle parallel to $y = x$ line, intercepts on some implicit equation have parallel tangents, etc.). Logarithmic differentiation and the second derivative in implicit differentiation is optional content, with no HW support.

Begin related rates. These are difficult for the student because they have to model a problem from scratch for the first time in the course (whichever word problem type is first, students find it hardest). Pythagoras type problems are nice level for this class, but be careful. If the triangle starts with no jog in it, then rates also satisfy the

Pythagorean identity. Most of the applications of related rates in this text involve just two variables changing, not a third. If you have time before the test, do 4.1, finding extrema of a continuous function on a closed interval. Important to include critical numbers for which f' dne. The text defines both relative (local) and absolute (global) extrema in this section, although the theory is only about absolute extrema.

Week 8 Test 2 in Week 8, no later (before drop date). This test is hardest for students because they reveal their limited algebra skills in computing derivatives and in using them.

Cover unit 3 in weeks 8.5–11 and make test 3 late week 11 or early week 12. There is a holiday in Fall term to work around, and spring break in spring term. Content is MVT (plus extrema on closed intervals if you didn't get there), optimization (TWO problems, one with the function given, one to model and solve), and curve sketching including asymptotes. Rolle's Theorem is not part of the course syllabus.

Weeks 9–10 The Mean Value Theorem should match the student's common sense: Somewhere the instantaneous rate of change will match the average rate of change. Computationally finding c from the MVT isn't what that theorem is about. You can present MVT geometrically, and focus on its power: application to curve sketching and how that informs extreme value computations, well-definedness of antiderivatives again match the student common sense, and speeding violations linked to timed toll road tickets (Indiana Toll Road and Ohio Turnpike are two such, and something in Florida). Only 1.5 lectures on MVT.

Then move on to how a graph can be determined from the signs of f' and f''. Be sure to show an example of a quartic where $f'' = 0$ does NOT give inflection (students have the wrong impression there) and spend two classes on this, including basic properties of polynomials (which they don't know: end behavior, (maximal) number of x-intercepts, (maximal) number of turnarounds, a root with a horizontal tangent is a multiple root, etc). Recitation is a good third class to set them to work together building the graphs.

Then 1.5–2 classes on all infinite limits and asymptotes (when the limit $= \infty$ and taking limits $x \to \infty$), including slant asymptotes.

Finish curve sketching with problems that work in reverse—given graphs of f' and/or f'', what can you tell about f? Also, given a point or two and sign charts for f' and f'' (or a table of info), can you build the graph of f? Finally, given a graph of velocity, what can you infer about the position function? These test their logic development and are perhaps pedagogically more defensible in the modern era where graphing devices are everywhere.

Week 11 This unit ends with Optimization, and you need a whole week, plus a quiz and review. The sign charts should help the student understand visually the process whereby local extrema can be found. The process for solving an optimization problem needs to be outlined, but you can do this by working through a problem and deriving the steps as you go along, then generalizing those steps after the solution. This text uses the terms "primary" and "secondary equations" and so stick with that. The difficult part for students is identifying the domain. A rule of thumb to help them is that the secondary equation gives info about the domain: let one variable shrink down and then the other to find the ends of the domain. Domain determines what kind of analysis should be done on the critical numbers: closed domain analysis was done in

4.1. Open domain analysis requires more argument: A relative extremum needs to be argued into an absolute extreme, which is easy if there is just one critical value. The text includes the Second Derivative Test: Remind students that it only tests *relative* extrema and does not establish *absolute* extrema. The First Derivative Test is better, and matches the sign chart work from the previous unit—some students do this analysis even when the domain is closed (an all-purpose test). Warning: The "watch it" feature in WebAssign on some optimization problems uses the Second Derivative Test or skips justification entirely, so emphasize in class that justification of absolute extrema is always necessary.

Week 12 You begin week 12 with test 3, or the end of week 11. Unit 4 is shorter, weeks 12.5–15.5 and a test. Differential approximations take 2 lectures (plus homework discussion time)—linear approximations first, then analyzing what differentials tell you. Some of the linearizations can be done with data rather than a given function; see the homework set. The word problem differential approximations depend on your view of elementary calculus for terminal students.

Shifrin does NOT like the (incorrect) use of differentials here. He likes to talk about the estimate using the tangent line, and the point of the tangent line is that relative error is smallest. Townsley loves: "The final computation should be within 2% of ideal, so how accurate should initial measurement be?" This section gets more word problems in the course syllabus.

Weeks 12–13 Appendix F is a review of limits, related rates, differential approximations, and optimization in a business context. This is a nice setup to prepare for the final exam. The HW set covers all of these ideas from across the text. The appendix is available in WebAssign as pdf (in Resources), and the first page is a lovely summary of the business terms. Notice that the text problems always have $x =$ demand. Hiding in the text are two theorems: maximum profit occurs when marginal cost = marginal revenue (see Figure F.6) and minimal average cost occurs when average cost = marginal cost (exercise 17 in the appendix). Discuss why these ideas make sense in the business context, and add in the mathematical meaning of the point of diminishing returns. The optimization examples should be three forms: one where you build a (linear) pricing function, then revenue and maximize either revenue or profit, a second to minimize cost or average cost, and a third type where there is a geometric problem from previously but there is cost attached weighting the various components (the bin above is made of two kinds of metal, base is \$20/ft^2 and sides are \$10/ft).

The unit ends with moderately easy work: antiderivatives and initial value problems. Applications are acceleration/velocity/position again—falling objects and braking cars. To learn antiderivatives, be sure to include examples where rewriting is necessary first, like $dy/dx = 5/(3t^2)$ and $dy/dx = (2x+3)(5x^2-1)$. Students need to be able to find antiderivatives of e^{mx} and $\sin(kx)$ (other trigs also) and $1/(mx+b)$. (The last will appear in ODE's that they are going to solve in a business setting to end the course.)

Weeks 14–15 You can put the last content, 6.2–6.3, either before or after test 4. Cover 6.2–6.3 solving variable separable differential equations and applications. Students should be able to solve ODE's involving polynomial solutions and use initial values appropriately. The ODE $dy/dt = ky$ (exponential growth and decay) and $dy/dt = k/(my+b)$ (Newton's Law of Cooling) or $dA/dt = rA \pm p$ (mortgages and savings plans) should show up on the final, both solving and applications (the application parts are actually at the precalc level). You will also need some review time for the final exam.

APPENDIX: Additional Problems Coded at UGA

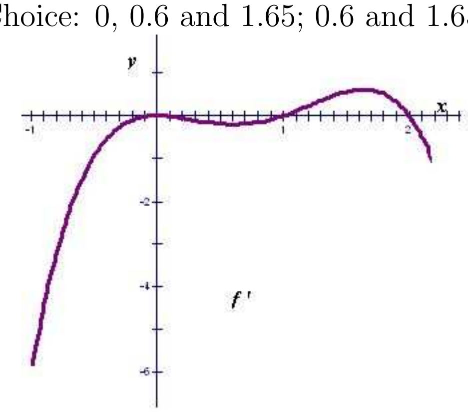
Below are some problems coded at UGA and the HW sets in which they appear in WebAssign. For example, 4.6.6 is the sixth problem in HW 4.6.

1.6.19 Find all solutions of the equation $e^{2x} - 10e^{x} + 25 = 0$

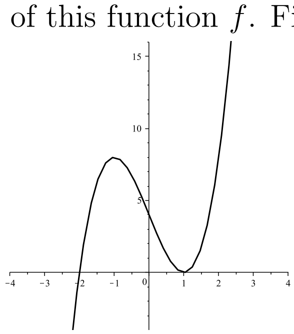
1.6.20 Solve the following equation for x: $\log_8(x) - \log_8(1-x) = 4$

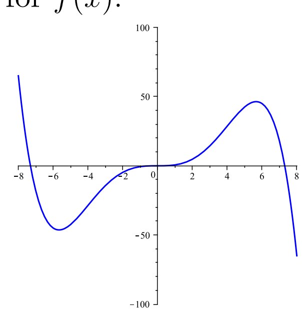
4.6.6 Below is a graph of f', the DERIVATIVE of the function f on domain $(-1, 2.2)$. Use the graph to answer the following questions about the function f:

- a On what interval(s) is the function f increasing?
  Multiple Choice: $(1, 2)$, $(-1, 0) \cup (0.6, 1.65)$, $(-1, 2.2)$
- b What are the critical numbers of f?
  Multiple Choice: 0, 0.65 and 1.65; 1 only; 0, 1 and 2
- c At what x-values does f have a relative minimum?
  Multiple Choice: 0.6, 1, 2
- d At what x-values does f have a point of inflection?
  Multiple Choice: 0, 0.6 and 1.65; 0.6 and 1.65; 0.2, 1



4.6.7 Below on the left is the graph of a degree 3 polynomial. The point $(2.5, 20.25)$ is on the graph of this function f. Find a formula for $f(x)$.





4.6.8 The graph of a polynomial is given on the right above. What is the least possible degree of this polynomial? [Multiple Choice: 2, 3, 4, 5, 6]

AppF.7 A price p (in dollars) and demand x for a product are related by $2x^2 - 2xp + 50p^2 = 7400$. If the price is increasing at a rate of 2 dollars per month when the price is 10 dollars, find the rate of change of the demand.

AppF.8 The profit P for a company is

$$P = 100xe^{-x/550},$$

where x is demand and P is measured in millions of dollars. At the instant that $x = 110$, demand is increasing at a rate of 7 units/month. Find the rate of change of profit at that instant.

AppF.15 A business (Widgets, Inc) forms a linear model of its widget pricing function based on the following information:

Let x represent the number of widgets sold, and $p(x)$ the price per widget in dollars. The firm begins by selling $x = 350$ widgets at a set price of $50 each. After holding a "sale", the firm proposes that a $10 discount on the price will yield an increase of 30 more widgets sold.

a Find the linear pricing function $p(x)$ based on this information.

b Based on the fact that the firm wants both sales and the price to be positive, what is the appropriate domain of the pricing function $p(x)$?

c What is the revenue function? (revenue is total sales income)

d What sales price p yields maximum revenue?

AppF.16 A commuter train carries 800 passengers each day from Macon to Atlanta. It costs 3 dollars per person to ride the train. Market research reveals that 40 more people would ride the train for each 25 cent decrease in the fare. Assume a linear relationship between the fare and the number of riders. What fare should be charged in order to collect the largest possible revenue?

5.1A.19 Find the indefinite integral and check your result by differentiation. (Use C for the constant of integration.)

$$\int \left(\sin(3x) + e^{-7x}\right) dx$$

5.1A.20 Find the indefinite integral and check your result by differentiation. (Use C for the constant of integration.)

$$\int (\sec(2x)\tan(2x))\, dx$$

5.1A.21 Find the indefinite integral and check your result by differentiation. (Use C for the constant of integration.)

$$\int \frac{dx}{2x - 5}$$