Sponsored by: UGA Math Department and UGA Math Club

Team Round / 1 hour / 210 points
October 15, 2011

No calculators are allowed on this test. You do not have to provide proofs; only the answers matter. Each problem is worth 70 points, for a total of 210 points.

Problem 1 (Divide and conquer)**.** Consider a regular tetrahedron and a sphere with the same centers. Into how many parts can they divide three-dimensional space? The sizes of the tetrahedron and of the sphere can be arbitrary.

Problem 2 (Deduction)**.** On a 5 by 5 grid, there are 25 squares, and in each square there is a number. For each little square, one computes the sum of the number in this square and the numbers in all the squares that share either a side or a vertex with it. This gives 25 new numbers.

This grid of sums is shown below. How many of the original 25 numbers can be recovered from this grid? In your answer, for each little square, either write down the original number, or put the letter U if the original number is impossible to compute.

18	12	8	10	16
5	1	6	13	17
-12	-4	1	15	7
-19	-19	-12	2	2
-6	-8	-10	-1	1

Problem 3 (Ackermann's boxes)**.** Four boxes, numbered from 1 to 4, initially contain one coin each. There are two moves that manipulate the number of coins in each box. The first move allows you to move one coin from box i to box $i+1$, if $1 \leq i < 4$; however, if you move a coin from box 3 to box 4, then a "bonus" coin appears in box 4 along with the one that was moved there. The second move allows you to remove one coin from box i and then switch the contents of boxes j and k, if $1 \leq i < j < k \leq 4$. Of course, at no moment can the number of coins in a box go negative.

What is the largest number of coins that the last box (number 4) could contain after a sequence of such moves?

Authors. All problems and solutions are written by Boris and Valery Alexeev.

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Team ID:

Team name:

Answer 1:

Answer 2:

Answer 3: