Topology Qualifying Exam

Aug. 14 2008

Justify all the calculations in your answers.

1. Let A be a proper subset of X and let B be a proper subset of Y. If X and Y are connected show that $(X \times Y) - (A \times B)$ is connected.

2. Recall that a topological space is said to be regular if every point p and every closed set F which does not contain p can be separated by open sets.

   (a) Prove that a subset of a regular space is regular.

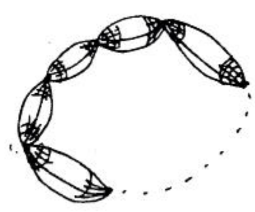
   (b) Give an example of a Hausdorff space which is not regular.

3. Let Y be $S^2 \times [0,1]$. Let X be the space obtained by cutting three tunnels out of this spherical shell; in other words, $X = Y \setminus \cup_{i=1}^{3} (D_i^2 \times [0,1])$, where the D_i are three small open discs in S^2. (The surface is shown below.) What is the genus of the boundary of X? Justify your answer.

4. Let X be the quotient space of the annulus obtained by identifying antipodal points on the outer circle and points on the inner circle which are 120 degrees apart. What is $\pi_1(X)$?

5. List all of the 3-fold covering spaces of Σ_2 (the closed orientable surface of genus 2). Explain why your list contains all possible 3-fold covers.

6. Compute π_1 and the homology of the the singular surface in the picture created by joining n "footballs" end to end in a ring. Each "football" meets the next in a single point.



7. Use the Mayer-Vietoris sequence to compute the homology groups $H_i(S^n)$ for all i and all $n > 0$.

8. Let $f : S^n \to S^n$ be a map without fixed points. What is the degree of f? Explain.

9. Let $\alpha_{pq} \subset S^1 \times S^1$ be the image of the line $y = \frac{q}{p}x \subset R^2$ under the standard projection $\mathbb{R}^2 \to S^1 \times S^1$ (given by $(x, y) \sim (x + m, y + n)$ for all $m, n \in \mathbb{Z}$. Calculate the homology of $(S^1 \times S^1)/\alpha_{pq}$.