ABSTRACT ALGEBRA:
A Geometric Approach

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Errata and Typographical Errors

p. 17, lines **3-4**. N is the smallest positive integer having .... Note, in particular, that N cannot be prime; now, suppose ...

p. 18, Exercise **3**. $\gcd(0, m) = |m|$.

p. 18, Exercise **13**. $\mu = |ab|$ and $|ab|/d$.

p. 21, line **4**. Delete ")".

p. 24, line **8**. Delete the second "p."

p. 35, Exercise **38**. Using $e = 101$, check that $d = 317$. In the first line of the table, 1663 should be 1633. ...Using the same N and $e = 1601$, ...

p. 39, line **14**. $\dots a^{-1}(ab) = a^{-1} \cdot 0 = 0 \dots$

p. 47, line **10**. Given $n \in \mathbb{Z}$, exactly one of the statements

$$n \in \mathbb{Z}^+, \; n = 0, \text{ or } -n \in \mathbb{Z}^+$$

is true. (This is called the trichotomy principle.)
line **15**. Z^+ should be $\mathbb{Z}^+$.
line **21**. Given $x \in F$, exactly one of the statements

$$x \in F^+, \; x = 0, \text{ or } -x \in F^+$$

is true.

p. 66, Exercise **14**. This is correct when $v \geq 0$. What's the answer when $v < 0$?

p. 66, Exercise **17**. Delete the first dx.

p. 91, lines **1–4**. $p_1(x), \dots, p_m(x)$ should be *distinct* irreducible polynomials. And $\deg(r_j(x)) < \deg(p_j(x)^{\nu_j})$ for $j = 1, \dots, m$.

p. 92, Exercise **9**. Parts a. and b. are much harder than I thought when I wrote the problem. Even deciding if there are any irreducible elements of $\mathbb{Z}_6[x]$ seems quite difficult.

p. 96, proof of Theorem 2.1. The penultimate sentence of the first paragraph should read: Thus whenever $|z| \geq R$, $|f(z)| \geq |z|^n(1 - \frac{1}{2}) \geq R^n/2$. At the end of the second paragraph, it should therefore be noted that z_o cannot lie on the boundary circle; this is needed to complete the contradiction at the end of the proof.

p. 116, line **3**. (ya) should be (ya^{-1}) at the end of the line.

p. 117, Example **3**, last line. $\langle 2 \rangle = \langle 4 \rangle \subset \mathbb{Z}_6$, and $4 = 2 \cdot 2$, so 4 is a multiple of 2 by a zero-divisor. On the other hand, $4 = -2$, so 4 is *also* a multiple of 2 by a unit! In fact, in $\mathbb{Z}_n$ it can be shown that $\langle a \rangle = \langle b \rangle \iff a = sb$ for some unit s. (**N.B.** One can construct a commutative ring R having elements a, b satisfying $\langle a \rangle = \langle b \rangle$ and yet $a = sb$ for *no* unit s!)

p. 118, line **–10**. Proposition 2.2 of Chapter 3.

pp. 123-4, Exercise **16**. A few symbols didn't print here:

a. Given an ideal $J \subset S$, define

$$\mathcal{J} = \{a \in R : \phi(a) \in J\} \subset R\,.$$

(This is usually denoted by $\phi^{-1}(J)$.) Prove that $\mathcal{J}$ is an ideal.

b. Given an ideal $I \subset R$, define

$$\mathcal{I} = \{\phi(a) : a \in I\} \subset S\,.$$

. . . Prove that $\mathcal{I}$ is an ideal, provided ϕ maps onto S. . . .

p. 128, last line: $\overline{3}$ should be 3.

p. 130, line **3** of the proof of Theorem 2.4. ev_α maps $F[x]$ onto $F[\alpha]$ and has . . . .

p. 132, line **5** of the proof of Corollary 2.6. The only polynomial $f(x) \in \mathbb{Q}[x]$ having c as a root . . .

p. 133, **Remark** and Corollary **2.8**. In the displayed statement marked with ($*$), we must assume that $\alpha \notin \mathbb{Q}$. In the application, we should point out that $\alpha \notin \mathbb{Q}$ (e.g., by applying the result of Exercise 2.2.11a).

p. 138, Exercise **28a**. We must assume $p \neq 0$.

p. 143, line **8**. $\psi(a + bi) = a + 3b \pmod 5$.

p. 163, line **4**. for some nonnegative integer r . . .

p. 184, lines **9–10**. . . . in $\mathbb{Z}_5^\times$ there is only one.

p. 194, lines **3–4**. That is, the conjugate of $F = F_1$ (which is the flip fixing vertex 1) by R is F_2 (which is the flip fixing vertex 2).

p. 258, line **8** of Proof of Theorem **5.8**. $k(aH) \subset aH$.

p. 260, Exercise **18**. Assume also that if $\alpha = 1$, then $m > 1$.

p. 265, **last line** "... extending ϕ (...) and carrying α to α'."

p. 266, line **10**. "If $\tilde{\phi}(p(\alpha)) = 0$, ..."

Proof of Proposition **6.4**. Assume $K \neq F$.

p. 268, line **4**. $F'[\alpha_j]$ should be $F'[\alpha'_j]$.

p. 269, line **8**. "splits in K."

p. 272, line **15**. $-\omega$ should be $\overline{\omega}$.

p. 275, line **8**. t should be ℓ.

p. 277, Exercise **5**. $\phi\psi^2$ should be $\psi\phi^2$.

p. 287, line −**8**. $|x_1y_2 - x_2y_2|$ should be $|x_1y_2 - x_2y_1|$.

p. 302, line **1**. $\{\infty\}$ should be $\mathbb{P}^1_\infty$.

p. 304, last line. The coordinates of B and C are reversed.

p. 308, Figure **11**. α, β, γ should be β, γ, and α, respectively.

p. 310–11. $t' = g(t) = -\frac{d+et}{a+bt}$, which makes the matrix $\begin{bmatrix} a & b \\ -d & -e \end{bmatrix}$ and $g^{-1}(t) = -\frac{d+at}{e+bt}$.

p. 316, Exercise **18**. This is false with the definitions provided. Indeed, we should really say that *six points are in general position provided no three are collinear and they do not all lie on a conic*. Then Exercise 18 becomes a triviality.

p. 319, line **6**. $\mathbf{x}_1, \dots, \mathbf{x}_5 \in \mathbb{R}^3$.

p. 324, line **4**. $\mathbf{v}_2 = \frac{1}{3\sqrt{2}}(1, -4, 1)$.

p. 324, line **−5**. $y_i = \sqrt{|\lambda_i|}x_i$.

p. 325, line **11**. $z_i = \sqrt{|\lambda_i|}y_i$.

p. 342, Exercise **20**. A few symbols didn't print here:

…Associate to the line $\overleftrightarrow{PQ}$ the point $\mathcal{A} = [a_{01}, a_{02}, a_{03}, a_{12}, a_{13}, a_{23}] \in \mathbb{P}^5$.

a. Prove that $\mathcal{A}$ is a well-defined point in $\mathbb{P}^5$ determined only by the line. (…the point $\mathcal{A} \in \mathbb{P}^5$ doesn't change. …$a'_{ij} = p'_i q'_j - p'_j q'_i$, we have $\mathcal{A}' = \mathcal{A} \in \mathbb{P}^5$.)

b. Show that $\mathcal{A}$ satisfies the (homogeneous) quadratic equation …

c. Given two lines ℓ and m, let $\mathcal{A}$ and $\mathcal{B}$ be the corresponding points in $\mathbb{P}^5$. …

p. 361, line **15**. "two" should be "four."

p. 363, Exercise **6**. The reference should be to Lemma 5.7.

p. 367, Exercise **30**b,c. The factors of $\frac{1}{2}$ should be 2's.

p. 371, line **6**. Q should be $\mathbb{Q}$.

p. 378, Exercise **5**. $h = g \circ f$; interchange f and g in parts a., b., c.

p. 387, Figure **1**. The labels P and P^{-1} should be interchanged.

p. 401, Exercise **1a**. …prove λ^n is an eigenvalue of T^n ….

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