University of Georgia   **Math 1113**

Department of Mathematics   **Test 2**   Spring 2019

By providing my signature below I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else during the exam:

Name (sign): _____   Name (print): _____

Student Number: _____

Instructor's Name: _____   Class Time: _____

- If you need extra space use the last page.

- Please show your work. **An unjustified answer may receive little or no credit.**

- If you make use of a theorem to justify a conclusion then state the theorem used by name.

- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.

- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.

- Please turn off your mobile phone.

- A calculator is not necessary, but numerical answers should be given in a form that can be directly entered into a calculator.

- Common identities:

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta),$$
$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta).$$

Problem Number	Points Possible	Points Made
1	0	
2	20	
3	20	
4	10	
5	15	
6	10	
7	15	
8	10	
Total:	100	

1. [2 Bonus] Common Knowledge: Who will win the 2020 Paris-Roubaix race?

2. Determine all of the values of x for each question below that satisfy the given equation.

   (a) [6 pts] $5^{2x} = 7$

   (b) [7 pts] $\ln(x) + \ln(x + 1) = 14$

   (c) [7 pts] $18 \cdot 3^{x-1} = 7 \cdot 8^{1-3x}$

out of a possible 20 points

3. For each part below a function is given. State if the function is exponential, linear, quadratic, logarithmic or something else. Also, state whether the function is always increasing, always decreasing, or neither.

   (a) [5 pts] $g(x) = 14^{-3x}$

   (b) [5 pts] $h(x) = \dfrac{3}{4 - x}$

   (c) [5 pts] $k(x) = \log_{10}(10 - x)$

   (d) [5 pts] $s(x) = 8x^2 - 4x + 10$

4. The questions below are related to the idea of determining when a function is one-to-one.

   (a) [7 pts] Show that the function $m(x) = \sqrt{3 - x}$ is one-to-one. (Do not just use a graph but demonstrate that the function is one-to-one algebraically.)

   (b) [3 pts] For what possible values of m is the function $p(x) = m \cdot x - 7$ one-to-one?

5. A trader is considering the purchase of a bond, but in the future the bond might default which means it will not be paid. The probability that a bond, bond #1, will default **after** some given time, t, is given by

$$e^{-2t}.$$

(This assumes that $t \geq 0$.)

(a) [6 pts] If the probability that a bond will default is $\frac{1}{2}$, how much time has passed?

(b) [6 pts] If the probability that a bond will default is $\frac{1}{4}$, how much time has passed?

(c) [3 pts] The trader is considering a second bond, bond #2, and the probability it will default **after** some given time, t, is given by

$$e^{-at},$$

and the value of a is a constant, **positive** number. What values of a will guarantee a **higher** probability of default after a given time and will be a more attractive bond to purchase?

6. [10 pts] A bank advertises a savings account which has an annual interest compounded weekly. (Assume that there are 52 weeks in a year.) The advertisement says that a person's initial investment will double in forty years. What is the annual compounded interest rate?

7. Due to the prevalence of high frequency trading people have developed mathematical models to approximate the expected time between trades of a given value. One approximation is the ACD model, and the expected time between trades (in milliseconds) whose value is x dollars is approximated by

$$-2.5 + \alpha \ln(x),$$

and the value of α is some constant number.

(a) [5 pts] If $\alpha = 0.3$ what is the expected time between trades of \$5,000?

(b) [5 pts] If the expected time between trades of \$3,000 is 0.2 milliseconds, what is the value of α?

(c) [5 pts] If it is determined that $\alpha \approx 0.4$, and the time between trades has to be positive, what is the domain of the function? (For what values of x is the expected time greater than zero?)

out of a possible 15 points

8. [10 pts] A radioactive material is found, and the initial mass of the material is approximately 10.5g. After twenty days there is approximately 8.4g of the material. How much material we there be after forty days?

Points earned: _____
out of a possible 10 points

Extra space for work. **Do not detach this page.** If you want us to consider the work on this page you should print your name, instructor and class meeting time below.

Name (print): _____ Instructor (print): _____ Time: _____