By providing my signature below, I acknowledge that I abide by the University's academic honesty policy. This is my work, and I did not get any help from anyone else:

Name (print): Solutions Name (sign): ____________

Student ID (81#): ____________

Instructor's Name: ____________ Class Time: ____________

Problem Number	Points Possible	Points Earned
1	16	
2	20	
3	10	
4	26	
5	18	
6	12	
7	24	
8	14	
9	12	
10	14	
Total:	166	

- If you need extra space use the last page. *Do not tear off the last page!*
- Please show your work. **An unjustified answer may receive little or no credit.**
- If you make use of a theorem to justify a conclusion, then state the theorem used by name.
- Your work must be **neat**. If I can't read it (or can't find it), I can't grade it.
- The total number of possible points that is assigned for each problem is shown here. The number of points for each subproblem is shown within the exam.
- Cell phones and smart watches are NOT allowed; smart devices (including smart watches and cell hones) may not be on your person and must be stored in a backpack, purse, or other storage item left at the front of the classroom.
- You are only allowed to use a **TI-30XS Multiview** calculator; the name must match exactly. No other calculators are permitted, and sharing of calculators is not permitted.
- You do not have to use a calculator; answers containing symbolic expressions such as $\cos(\pi/3)$ and $\ln(e^4)$ are acceptable. Include an exact answer for each problem.

1. Determine the following limits; briefly explain your thinking on each one. If you apply L'Hopital's rule, indicate where you have applied it and why you can apply it. If your final answer is "does not exist," ∞, or $-\infty$, briefly explain your answer. (You will not receive full credit for a "does not exist" answer if the answer is ∞ or $-\infty$.)

_____ (a) [4 pts] $\lim_{x\to 2} \left(x^3 - 7x + 17\right) = 8 - 14 + 17 = \boxed{11}$

_____ (b) [6 pts] $\lim_{x\to 0} \frac{2x}{e^{3x} - 7x - 1} \overset{LH}{=} \lim_{x\to 0} \frac{2}{3e^{3x} - 7} = \frac{2}{3-7} = \boxed{-\frac{1}{2}}$

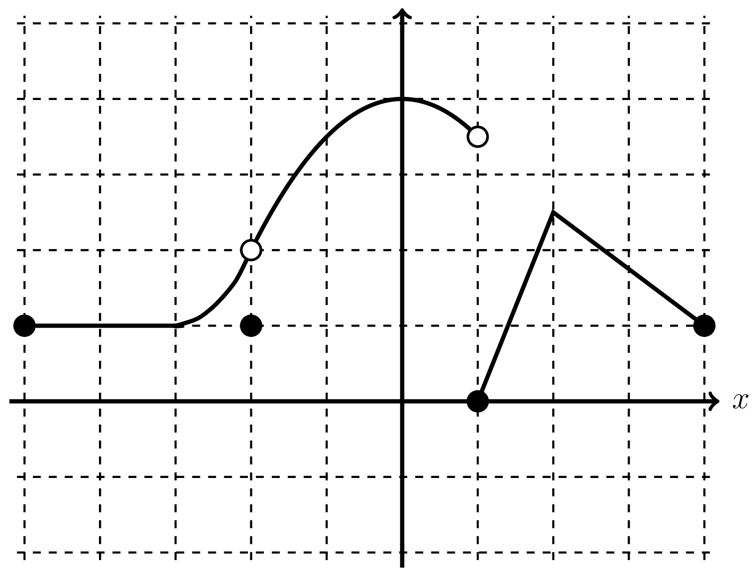
IF $\frac{0}{0}$

_____ (c) [6 pts] $\lim_{x\to\infty} \frac{\ln(x)}{7x^2} \overset{LH}{=} \lim_{x\to\infty} \frac{(1/x)}{14x} = \lim_{x\to\infty} \frac{1}{14x^2} = \boxed{0}$

IF $\frac{\infty}{\infty}$

numerator is 1,
denominator increases without bound

2. Consider the graph of $y = f(x)$ given below. The grid lines are one unit apart. Based on the graph, answer the following questions.



(a) [4 pts] Determine $\lim_{x \to -2} f(x)$.

2

(b) [4 pts] Determine $f(1)$.

0

(c) [6 pts] Determine all values of x in the interval $(-5, 4)$ at which $f'(x) < 0$.

$(0,1) \cup (2,4)$

(d) [6 pts] Determine all values of x in the interval $(-5, 4)$ at which $f'(x)$ is undefined. Briefly explain your thinking.

$\underbrace{x=-2 \ , \ x=1}_{\text{discontinuities}} , \underbrace{x=2}_{\text{corner on graph}}$

3. (a) [2 pts] State the limit definition of the derivative of $f(x)$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

(b) [8 pts] Use the **limit definition of the derivative** to determine the derivative of $f(x)=\dfrac{1}{1-3x}$. No points will be awarded for the application of differentiation rules (and L'Hopital's rule is not allowed).

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{1-3(x+h)}-\frac{1}{1-3x}}{h}$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{1-3x}{(1-3x-3h)(1-3x)}-\frac{1-3x-3h}{(1-3x-3h)(1-3x)}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left(\frac{1-3x-1+3x+3h}{(1-3x-3h)(1-3x)}\right)$$

$$=\lim_{h\to 0}\frac{1}{\cancel{h}}\cdot\left(\frac{3\cancel{h}}{(1-3x-3h)(1-3x)}\right)$$

$$=\lim_{h\to 0}\frac{3}{(1-3x-3h)(1-3x)}$$

$$=\frac{3}{(1-3x)^2}$$

– or –

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$=\lim_{x\to a}\frac{\frac{1}{1-3x}-\frac{1}{1-3a}}{x-a}$$

$$=\lim_{x\to a}\frac{1}{x-a}\left(\frac{1-3a}{(1-3x)(1-3a)}-\frac{1-3x}{(1-3x)(1-3a)}\right)$$

$$=\lim_{x\to a}\frac{1}{x-a}\left(\frac{1-3a-1+3x}{(1-3x)(1-3a)}\right)$$

$$=\lim_{x\to a}\frac{1}{x-a}\left(\frac{3x-3a}{(1-3x)(1-3a)}\right)$$

$$=\lim_{x\to a}\frac{1}{\cancel{x-a}}\cdot\frac{3\cancel{(x-a)}}{(1-3x)(1-3a)}$$

$$=\frac{3}{(1-3a)^2}$$

4. Determine the first derivative of each of the following functions. Remember to use correct notation to write your final answer.

(a) [6 pts] $f(x) = \dfrac{x^5}{3} - \dfrac{1}{\sqrt{x}} + \pi^7$

$f'(x) = \frac{5}{3}x^4 + \frac{1}{2}x^{-3/2}$

(b) [6 pts] $f(x) = \dfrac{\sin(x)}{3x+4}$

$f'(x) = \dfrac{(3x+4)\cos(x) - \sin(x)(3)}{(3x+4)^2}$

(c) [6 pts] $f(x) = [x^2 + \arcsin(x)]^5$

$f'(x) = 5(x^2 + \arcsin(x))^4 \cdot \left(2x + \dfrac{1}{\sqrt{1-x^2}}\right)$

(d) [8 pts] $y = x^{\cos(x)}$

$\ln(y) = \ln(x^{\cos(x)})$

$\ln(y) = \cos(x)\ln(x)$

$\frac{1}{y}\frac{dy}{dx} = \cos(x) \cdot \frac{1}{x} + \ln(x) \cdot -\sin(x)$

$\frac{dy}{dx} = y\left(\frac{\cos(x)}{x} - \sin(x)\ln(x)\right)$

$\frac{dy}{dx} = x^{\cos(x)}\left(\frac{\cos(x)}{x} - \sin(x)\ln(x)\right)$

$y = e^{\ln(x^{\cos(x)})} = e^{\cos(x)\ln(x)}$

$\frac{dy}{dx} = e^{\cos(x)\ln(x)}\left[\frac{\cos(x)}{x} + -\sin(x)\ln(x)\right]$

– or –

5. Determine the following indefinite integrals.

(a) [6 pts] $\int \left(x^5 + 2\sqrt{x} - e^x\right)\, dx$ $= \frac{1}{6}x^6 + \frac{4}{3}x^{3/2} - e^x + C$

(b) [6 pts] $\int \left(\frac{3}{x} + \frac{4}{1+x^2}\right)\, dx$ $= 3\ln|x| + 4\arctan(x) + C$

(c) [6 pts] $\int \sin(x)e^{3\cos(x)}\, dx$ $= -\frac{1}{3}\int \underbrace{(-3\sin(x))\,e^{\overbrace{3\cos(x)}^{u}}\,dx}_{du} = -\frac{1}{3}\int e^u\, du = -\frac{1}{3}e^u + C$

$u = 3\cos(x)$

$du = -3\sin(x)dx$

$= \boxed{-\frac{1}{3}e^{3\cos(x)} + C}$

6. Evaluate the following definite integrals.

(a) [6 pts] $\int_{\pi/12}^{\pi/6} (\cos(3x))\,dx = \frac{1}{3}\sin(3x)\Big|_{x=\pi/12}^{x=\pi/6} = \frac{1}{3}\sin(\pi/2) - \frac{1}{3}\sin(3\pi/12)$

$= \frac{1}{3}\cdot 1 - \frac{1}{3}\cdot\frac{1}{\sqrt{2}} = \boxed{\frac{1}{3}\left(1-\frac{1}{\sqrt{2}}\right)}$

(b) [6 pts] $\int_0^1 (1-2x)^6\,dx = -\frac{1}{2}\int_0^1 (\underbrace{1-2x}_{u})^6 \cdot \underbrace{-2\,dx}_{du} = -\frac{1}{2}\int_1^{-1} u^6\,du = -\frac{1}{2}\cdot\frac{1}{7}u^7\Big|_{u=1}^{u=-1}$

$u = 1-2x$
$du = -2dx$
$x=0 \Rightarrow u=1$
$x=1 \Rightarrow u=-1$

$= -\frac{1}{14}(-1)^7 - -\frac{1}{14}(1)^7 = \frac{1}{14}+\frac{1}{14} = \frac{2}{14} = \boxed{\frac{1}{7}}$

7. An object moves along a straight line with acceleration function $a(t) = -10t$ meters per second squared, where $t \geq 0$. The object's initial velocity is 70 meters/second. Give appropriate units with each answer.

(a) [6 pts] Determine the velocity function $v(t)$.

$v(t) = \int -10t \, dt = -5t^2 + C$

$70 = -5(0)^2 + C$

$C = 70$

$\boxed{v(t) = -5t^2 + 70 \text{ meters per second}}$

(b) [6 pts] When is the particle at rest?

$0 = -5t^2 + 70$

$5t^2 = 70$

$t^2 = 14$

$t = \pm\sqrt{14}$ $* \, t \geq 0$

$\boxed{t = \sqrt{14} \text{ seconds}}$

(c) [6 pts] When is the object's velocity 50 meters per second?

$-5t^2 + 70 = 50$

$-5t^2 = -20$

$t^2 = 4$

$t = \pm 2$ $* \, t \geq 0$

$\boxed{t = 2 \text{ seconds}}$

(d) [6 pts] What is the displacement of the object from $t = 0$ seconds to $t = 5$ seconds?

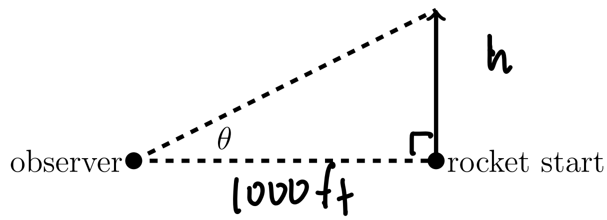
$\int_0^5 v(t)dt = \int_0^5 (-5t^2 + 70)dt = -\frac{5}{3}t^3 + 70t \Big|_{t=0}^{t=5}$

$= \frac{-5}{3} \cdot 5^3 + 70 \cdot 5$

$= \frac{-625}{3} + 350$

$= \frac{425}{3}$ ft

8. [14 pts] A rocket sitting on the ground is launched vertically upward and is tracked by an observer who is sitting on the ground 1000 feet from the launch site. Determine the velocity of the rocket when the angle of observation θ from the observer to the rocket is $\pi/6$ radians and is increasing at a rate of 0.2 radians per second.



goal: $\frac{dh}{dt}$

when $\theta = \pi/6$

$\frac{d\theta}{dt} = 0.2$

$\tan\theta = \frac{h}{1000}$

$1000 \tan\theta = h$

$1000 \left(\sec^2\theta\right) \frac{d\theta}{dt} = \frac{dh}{dt}$

$\sec^2\left(\frac{\pi}{6}\right) = \frac{1}{\cos^2\left(\frac{\pi}{6}\right)} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3}$

$1000\left(\frac{4}{3}\right)(0.2) = \frac{dh}{dt}$

$\frac{dh}{dt} = \boxed{\frac{800}{3} \text{ ft/sec}}$

— or —

$\theta = \arctan\left(\frac{h}{1000}\right)$

$\frac{d\theta}{dt} = \frac{1}{1 + \left(\frac{h}{1000}\right)^2} \cdot \frac{1}{1000} \cdot \frac{dh}{dt}$

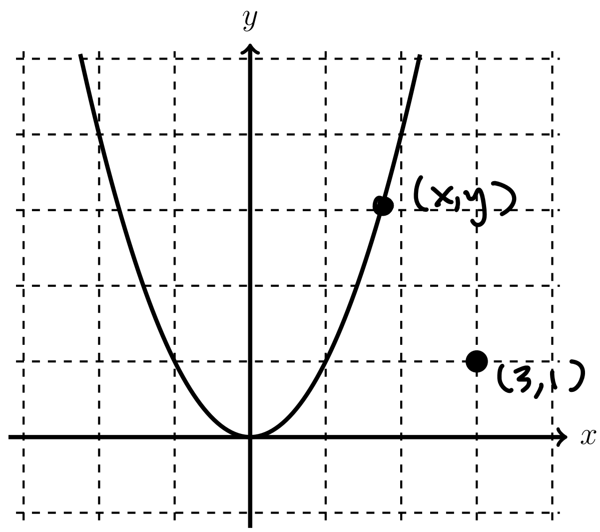
so $\frac{dh}{dt} = 1000\left(1 + \left(\frac{h}{1000}\right)^2\right) \frac{d\theta}{dt}$

$\frac{\pi}{6} = \arctan\left(\frac{h}{1000}\right)$

$\tan\left(\frac{\pi}{6}\right) = \frac{h}{1000}$ so $h = 1000\tan\left(\frac{\pi}{6}\right) = \frac{1000}{\sqrt{3}}$

$\frac{dh}{dt} = 1000\left(1 + \frac{1}{3}\right)(0.2) = \boxed{\frac{800}{3} \text{ ft/sec}}$

9. Consider the point $(3, 1)$ and the parabola $y = x^2$ graphed below. (Adjacent grid lines are one unit apart.)



_______ (a) [6 pts] Let (x, y) represent a point on the parabola $y = x^2$. Determine a function $f(x)$ for the distance from the point (x, y) to the point $(3, 1)$. Your function should include the variable x and can not include any other variables.

$$d = \sqrt{(x-3)^2 + (y-1)^2} \quad , \quad y = x^2$$

$$f(x) = \sqrt{(x-3)^2 + (x^2-1)^2}$$

_______ (b) [6 pts] Suppose you want to determine the closest point on the parabola to the point $(3, 1)$. Determine an appropriate domain for the function $f(x)$, and briefly explain your answer. (Do not determine the closest point.)

$(-\infty, \infty)$ - any pt on parabola — or

$(1, 3)$ - looks like closest point is in this interval — or

$(1, 2)$ - " " " — or

(lots of correct answers)

10. [14 pts] You plan to make an open-top box with a square base; you want to construct the box to have volume 5 ft^3 and the minimum possible cost. The cost of the material for the base of the box is \$3 per square foot, and the cost of the material for the sides of the box is \$1 per square foot.

Let w represent the (base) width of the box, respectively. Then the cost (in dollars) of the box is

$$C(w) = 3w^2 + \frac{20}{w}.$$

Using the function $C(w)$ with domain $(0, \infty)$, determine the width w that results in the minimum possible cost for your box. You must use calculus techniques to verify that your w results in the minimum possible cost.

$$C'(w) = 6w + \frac{-20}{w^2} = \frac{6w^3 - 20}{w^2}$$

critical pts: $6w^3 - 20 = 0$

$$6w^3 = 20$$
$$w^3 = \frac{20}{6} = \frac{10}{3}$$
$$w = \sqrt[3]{\frac{10}{3}} \quad (w > 0)$$

1st derivative test:

$\left(0, \sqrt[3]{\frac{10}{3}}\right)$	$\left(\sqrt[3]{\frac{10}{3}}, \infty\right)$
$C'(1) = -14$	$C'(3) = \frac{6 \cdot 27 - 20}{9} > 0$
dec	inc

2nd derivative test:

$$C''(w) = 6 + \frac{40}{w^3}$$

so $C''\left(\sqrt[3]{\frac{10}{3}}\right) > 0$

$$\boxed{\text{C has a local min at } w = \sqrt[3]{\tfrac{10}{3}} \text{ ft. Since } \sqrt[3]{\tfrac{10}{3}} \text{ is the only critical number, the min is absolute.}}$$

This page is extra space for work. **Do not detach this page.** If you want us to consider the work on this page, you should print your name, instructor and class meeting time below. For the problems where you want us to look at this work, please write "see last page" next to your work on the problem page so that we know to look here.

Name (print): ______________________ Instructor (print): ______________________

Class Meeting Time: ______________