

On Teaching:

"Nothing teaches like experience", John Bunyan.

Webster's dictionary (copyright 1943, The World Publishing Company),
Definitions and Synonyms for To Teach (lit. to show):

1. to enlighten (illuminate, enable to see truth, lit. shed light on),
2. to educate (nurture, develop, initiate, lit. draw forth),
3. to guide in studies,
4. to instruct (inform, admonish, lit. pile up),
5. to exhibit so as to impress on the mind,
6. indoctrinate (imbue someone else with one's own ideas),
7. train (drill; exercise; break, tame and reduce to docility, as to train dogs or monkeys; lit. to draw, pull).

Note that "to inspire" and "to increase interest in" are not among Webster's meanings for "to teach". Moreover in 2-7, even truth plays no role; one can teach evil, falsehood, and sloth.

Rather than use such language as "good teacher", [footnote: cf. the caveat in the Gospel according to St. Luke, chapter 18, verses 18-19] I prefer to use descriptive terms. I suggest that to inspire the students, to continually seek fresh insight and look for ways to explain it more clearly and vividly, to insist on critical thinking and high standards, and to exhibit in oneself a continuing thirst for knowledge, are the most desirable qualities of a teacher.

Among my own heroes of teaching, I consider Maurice Auslander, at Brandeis University, one of the finest teachers I have ever known. Although he was persistently criticized for lack of organization, his classroom exploded with spontaneity and excitement. His was dynamic teaching which was hard to follow, but which gave us powerful precepts (instead of details) and made us want to rush out and apply them, teaching which made our powers of comprehension and problem solving more trenchant.

He was clearly a powerful mathematician who was letting us watch him do mathematics, and showing us how to do it too. I felt Auslander was teaching us that mathematics is exciting, challenging and fun, that it deserves the best we have to give, and will repay the greatest effort we can muster.

I was drawn into my present specialty of algebraic geometry, by Alan Mayer, an extremely strong and imaginative geometer at Brandeis, whose lectures were so rapid-fire, so jam-packed with information, so lengthy and wide ranging, that I sometimes got headaches, and felt a need to recuperate after them for an hour in the sauna! They also made the subject so fascinating that there was nothing for me to do but to try to join the company of algebraic geometers.

Others among the best teachers I had in that field, Herb Clemens (Utah), David Mumford and Phillip Griffiths (Harvard), sometimes gave me instruction simply by expressing ignorance of a topic and saying that they wished somebody would explain it to

them. I of course rushed off to learn it and become the first to teach it to them. When I had spent days or weeks mastering it, they never seemed to need more than a few minutes to catch up and go well beyond what I had told them.

I admit that as a professional, in constant need of up-to-the-minute insights, I no longer scorn pedestrian explanations of useful information. But I still find it difficult to sit through such a lecture if I do not understand why the information is useful. So for me motivation is still necessary.

If a concept is defined by how it is measured, our teaching evaluation forms suggest that good teaching can be appreciated by an average student in the class who has not yet even finished the course. I cannot fit my own great teachers into this paradigm. Take Raoul Bott, who was regarded as an outstanding teacher at Harvard. Once, in his class on algebraic topology, he remarked after proving the Brouwer fixed point theorem that all we had really needed was a "homotopy invariant functor that doesn't vanish on the sphere." I did not follow this remark. I did not even grasp what it meant much less why it would prove the theorem. The theorem had a lengthy proof, and I did not understand how he could pretend to summarize it in one phrase. Since he did not write his comment down, I even remembered it wrong as a "homotopy invariant functor that vanishes on the sphere."

We might say, that Bott indulged himself in making deep, succinct statements even though the statement was not comprehensible to the class at large. Is this a good quality or a bad one? You may feel the answer depends on how many in the class find the statement comprehensible: the more the better. But I suggest that this behavior of Bott's is valuable teaching even if not a single student understands the statement! In fact it is more valuable to the student who does not understand it, because that student is being helped the most.

That student has already been taught all he/she can take at the moment, and is being pointed to higher ground which he/she will eventually be able to tread. That student is receiving instruction not just for the moment, but also for the future; hence being given something to think about which will last a significant amount of time, and which will repay all the thought given to it.

In my case, several years passed before I understood Bott's statement. It occurred when I began to appreciate the difference between building a tool and using it, between definitions and existence theorems. The details that had obscured my vision were the nuts and bolts of constructing the tool, and Bott's lightning summary contained only the key features of the finished tool. As I finally understood his comment I savored the knowledge in it, his generosity in saying it, and the satisfaction of resolving a puzzle of many months standing.

How often does one encounter the grateful comment on teaching evaluations "He really gave us provocative questions to think about. I still have not settled them all!"? If this comment is missing, can the teacher really be excellent? When I took Freshman calculus from John Tate (at Harvard) his lectures were very dynamic, his course was very

difficult, he knew instantly the answer to any question, and he could prove any statement in full detail apparently without a moment's reflection. But I did not know if he was a good teacher.

At Christmas I compared notes with a friend who had gone to a well known engineering school in the South. It was immediately clear that calculus was not the same everywhere, that my course was much more demanding, and that I was being given far more by Tate than my friend was getting from his professor. I began to realize that Tate was a good teacher.

The quality of a teacher was often measured by my undergraduate acquaintances, according to the quality and depth of the material being presented. The professor was praised for possessing a distinguished vision of the subject as much as, or more than, for a facility at making it easy. It was understood that deep material cannot be made easy. In Bott's course, for instance, his proof of the homotopy lifting property of covering spaces was sketchy and incomplete, and he seemed not to have any interest in writing out the details.

However I had no trouble finding it in every book on the subject, and eventually in working it out for myself. What I could not get for myself was the grasp of the big picture, the sense that it was possible to view all these things from a perspective from which they were quite trivial, and the inspiration to achieve that perspective. A teacher can be considered good in that sense only if she gives you something beyond what is in the books, and perhaps insists that you try to understand it. How often does one encounter comments like the following in a teaching evaluation:

"She really made this course hard by including points of view more sophisticated than those of the book, which appeared mundane in comparison. I have grown intellectually more in this quarter than ever before! Great teacher! The students last year were shortchanged by a professor who plodded through the syllabus, assigning only the easier problems."

One of the moments I remember best in a course by David Mumford was when he turned to the class and remarked, "the way to read Grothendieck is to find the topic you want, read that section (tracing back through the pages for all the references) and understand it, then go home and write it up yourself in two pages." This advice on how to extract information from tedious and lengthy source material is invaluable to the student who thinks he must slog through every book from the beginning.

How shall we progress beyond the minimal teaching skills associated with training people, displaying information, and instructing from a syllabus, to the deeper, more valuable ones of guidance, nurturing, illumination, and inspiration? I suggest we begin by emphasizing that these latter qualities are more important than simple information transferral. Steve Sigur, mathematics teacher at The Paideia School in Atlanta, has asserted that there is no point in teaching only for factual content, since after one year essentially no factual content is retained!

The truth of this brutal claim is evident to every teacher faced with verifying the prerequisites in a new class. Indeed the lifetime of "learned" information often seems only weeks or days after the final, instead of one year. Therefore I suggest that advice to

a new teacher include a reminder to volunteer to teach a variety of courses and to attend seminars and professional meetings, so that one's ability to inspire, enlighten, and draw connections between different topics continues to mature.

I agree with David Penney's warning that good teaching is zen-like; that there are as many potential manifestations of good teaching as there are teachers, and that we should each make use of our individual strengths. I agree too with John Hollingsworth, that the success of the teacher in accomplishing her goal is more important than the means, and we should measure teaching by observing the subsequent trajectory of the students, rather than by exit polls from the course.

He tells a story of a kangaroo court held at a well known state school to review complaints about the teaching of a certain famous but eccentric mathematics faculty member. The faculty member began by dividing the board in half by a line and writing a large number of names in one half. He was asked why. The response was that he felt it important that before reviewing his teaching they establish their respective credentials, so he had listed on one side of the board the names of those among his former students who had been elected to the National Academy of Sciences. The other half of the board was empty to allow his inquisitors to do the same. The group quickly voted to dismiss the hearing.

I would add that a good teacher's goal must be one worthy of pursuit. I would even argue (to paraphrase Randall McMurphy from *One Flew Over the Cuckoo's Nest*) that to have tried and failed to accomplish a worthy goal, even one which was plainly impossible, is deserving of admiration. For those of us who lament that we do not fully achieve our goals with all our students, I recommend the fourth chapter of the Gospel According to Saint Mark, where Jesus explains the importance of prerequisites, (although, to his dismay, not clearly enough for his disciples).

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